Quantum Teleportations on Trapped Ions:
~From Basics to Recent Research~

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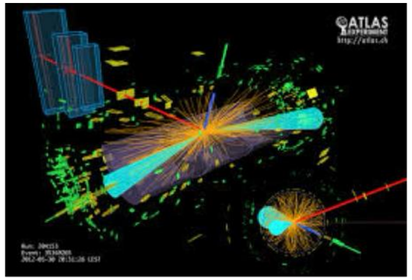
Seminar@東京女子大学理論物理研究室

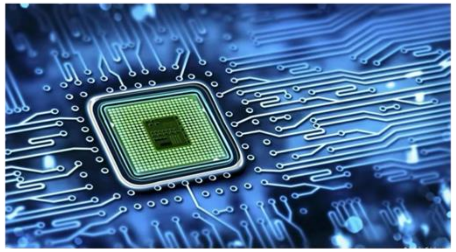
複雑な計算タスク

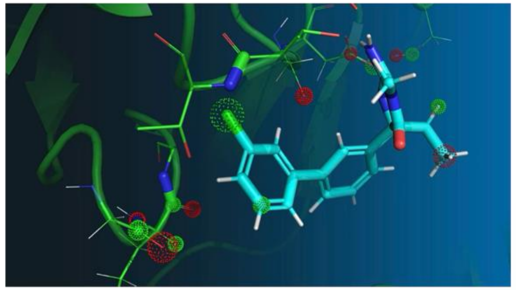
The new frontiers of computation



自然科学

物質科学





化学・創薬

組合せ最適化・AI

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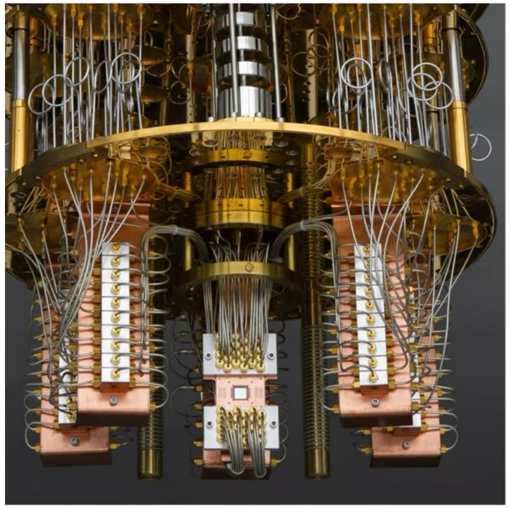
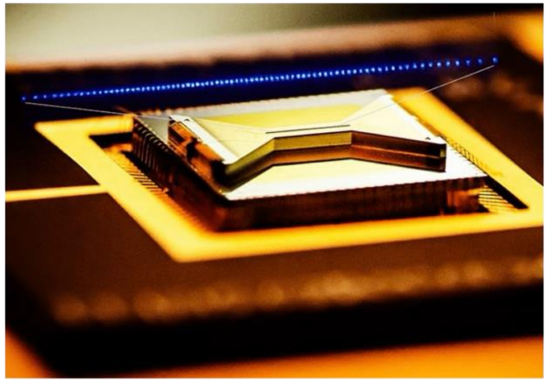
超伝導型



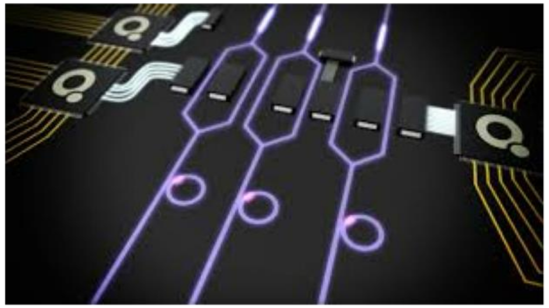
IBM, Google, 理研, …

イオントラップ



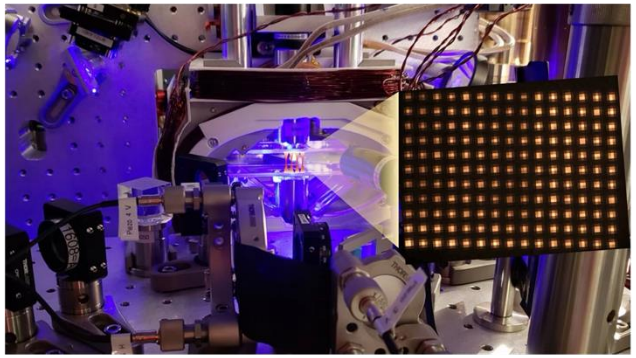
Quantinuu, IonQ, AQT …

光量子型



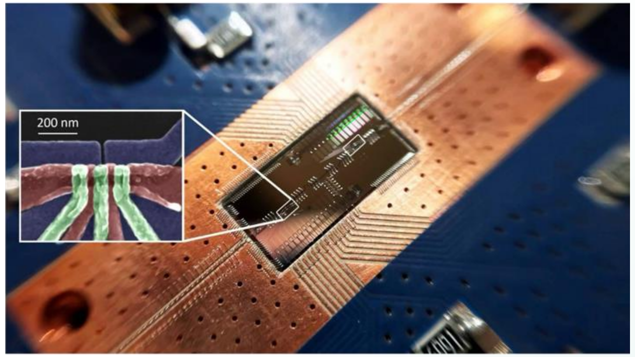
PsiQuantum, Xanadu, Quandela, …

中性原子（冷却原子）



QuEra, Pasqal…

スピン (シリコン)型



Intel, 日立, Diraq,…

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2. 最近の量子計算機を使った研究　～多体量子テレポーテーション～

Kazuhiro Seki, Y.K., Tomoya Hayata, Seiji Yunoki, arxiv:2405.07613

量子コンピュータ(ビットと量子ビット)

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(古典)ビット:(古典)コンピュータで扱う基本単位

- nビットはn個の0または1の並び。2^n 通りある0と1の組合せの列を表現できる

例. $011000010100 \ldots 01$

量子ビット:量子コンピュータで扱う基本単位

- 0と1が重なり合った状態を表現できる

$$\alpha|0\rangle + \beta|1\rangle, \quad |\alpha|^2 + |\beta|^2 = 1$$

- n量子ビットは2^n通りの0と1の組合せの列が重なり合った状態を表現できる

$$\alpha_1|00 \ldots 00\rangle + \alpha_2|00 \ldots 01\rangle + \alpha_3|00 \ldots 10\rangle + \ldots + \alpha_{2^n-1}|11 \ldots 10\rangle + \alpha_{2^n}|11 \ldots 11\rangle$$

ゲート式量子コンピュータ（論理ゲート）

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古典コンピュータは論理ゲートAND、OR 、NOTを組み合わせて、与えられたビット列から目的のビット列を与えることが可能

ユニバーサル量子ゲートセット

与えられた量子ビットから目的の量子ビットを与えるための基本操作（量子ゲート（ユニタリー行列））が存在

以下のゲートセットによって任意のユニタリー行列を表現できる

$$\mathrm{H} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix},$$

Hadamardゲート

$$\mathrm{CX} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix},$$

CNOTゲート

$$\mathrm{R}_Z(\theta) = \begin{pmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{pmatrix}$$

Z回転ゲート

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad |00\rangle = |0\rangle \otimes |0\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad |01\rangle = |0\rangle \otimes |0\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \ldots$$

ゲート式量子コンピュータ（量子ゲート）

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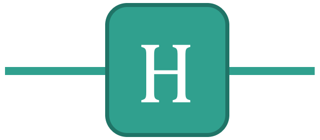
量子ゲートの作用の仕方

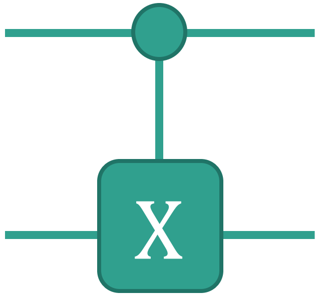
$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

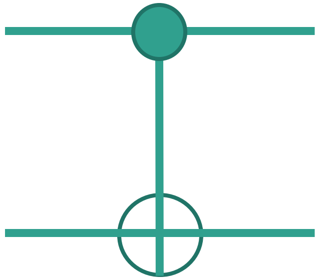


$$CX|00\rangle = |00\rangle \qquad CX|10\rangle = |11\rangle$$

$$CX|01\rangle = |01\rangle \qquad CX|11\rangle = |10\rangle$$



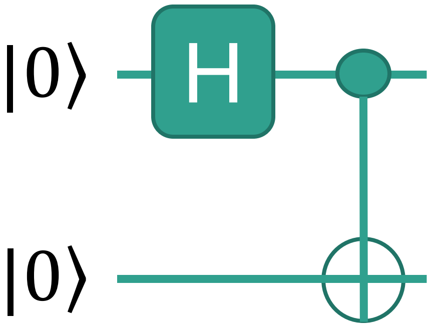
or



$$R_Z(\theta)|0\rangle = e^{-i\theta/2}|0\rangle$$

$$R_Z(\theta)|1\rangle = e^{+i\theta/2}|0\rangle$$

ゲート式量子コンピュータ（エンタングルメント状態）



Bell（EPR）状態

$$\mathrm{CX}\cdot(\mathrm{H}\otimes I)\cdot(|0\rangle\otimes|0\rangle)=\mathrm{CX}\left(\frac{|00\rangle+|10\rangle}{\sqrt{2}}\right)=\frac{|00\rangle+|11\rangle}{\sqrt{2}}$$

$$=\frac{|\psi^*\psi\rangle+|\bar{\psi}^*\bar{\psi}\rangle}{\sqrt{2}}$$

$$U|0\rangle=|\psi\rangle$$
$$U|1\rangle=|\bar{\psi}\rangle$$

どんなに頑張っても二つの1量子ビット状態のテンソル積

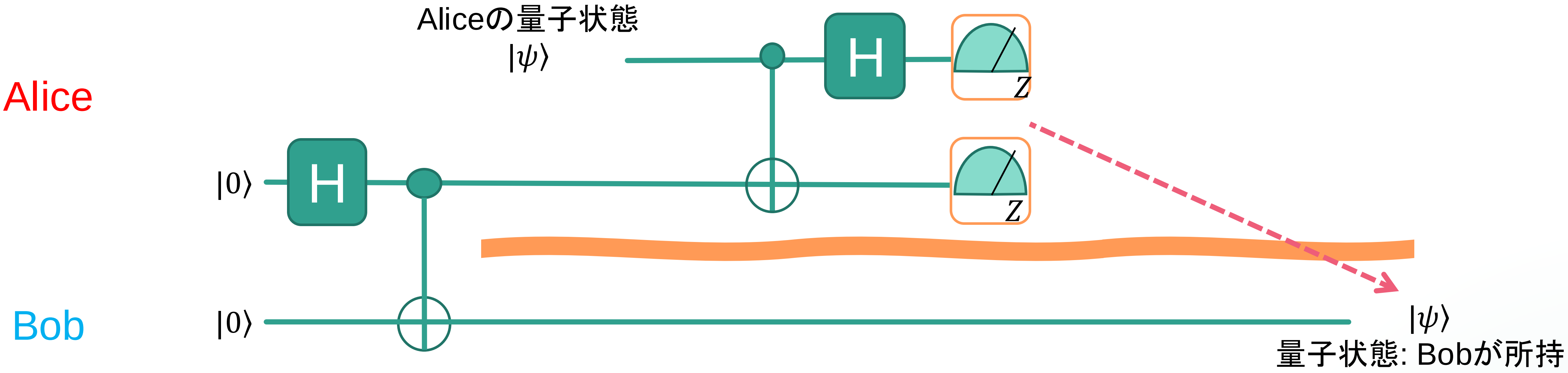
$$(\alpha_1|0\rangle+\beta_1|1\rangle)\otimes(\alpha_2\,|0\rangle+\beta_2|1\rangle)$$

の形（直積状態）で表現できない。

このように単純なテンソル積で表現できないものをエンタングル状態と呼ぶ。

応用として、このエンタングル状態を利用した量子テレポーテーションを紹介する。

量子テレポーテーション



量子テレポーテーション:エンタングル(Bell)状態と(Bell)測定を用いて、Alice が保持している量子ビットの量子状態をBob が保持している量子ビットに転送できる。

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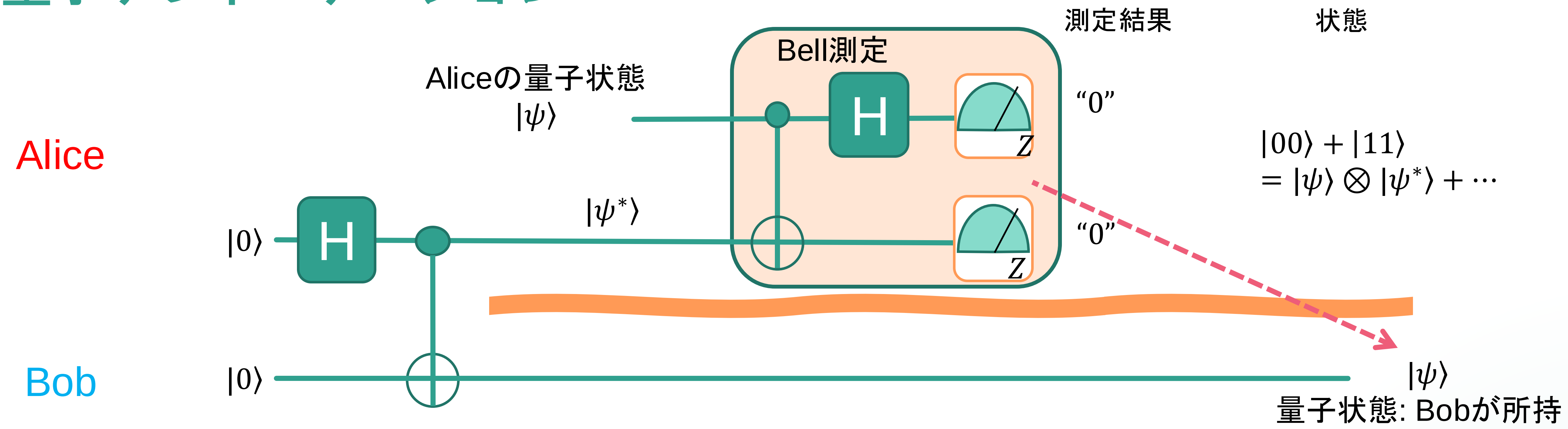
量子テレポーテーション



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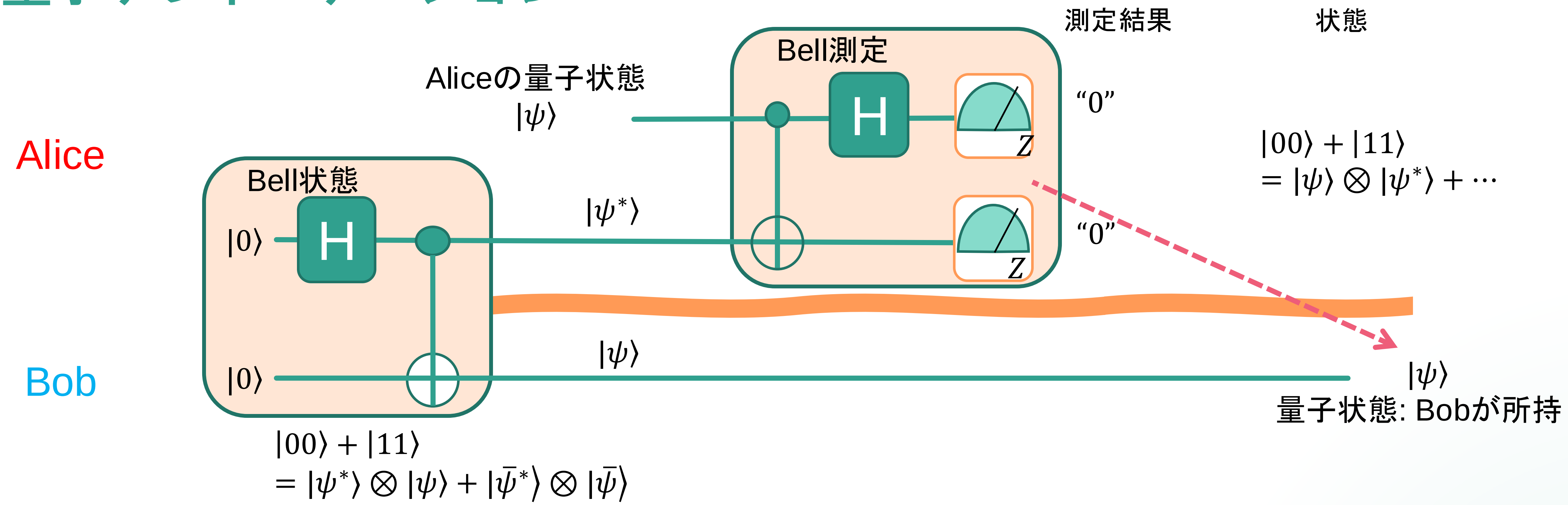

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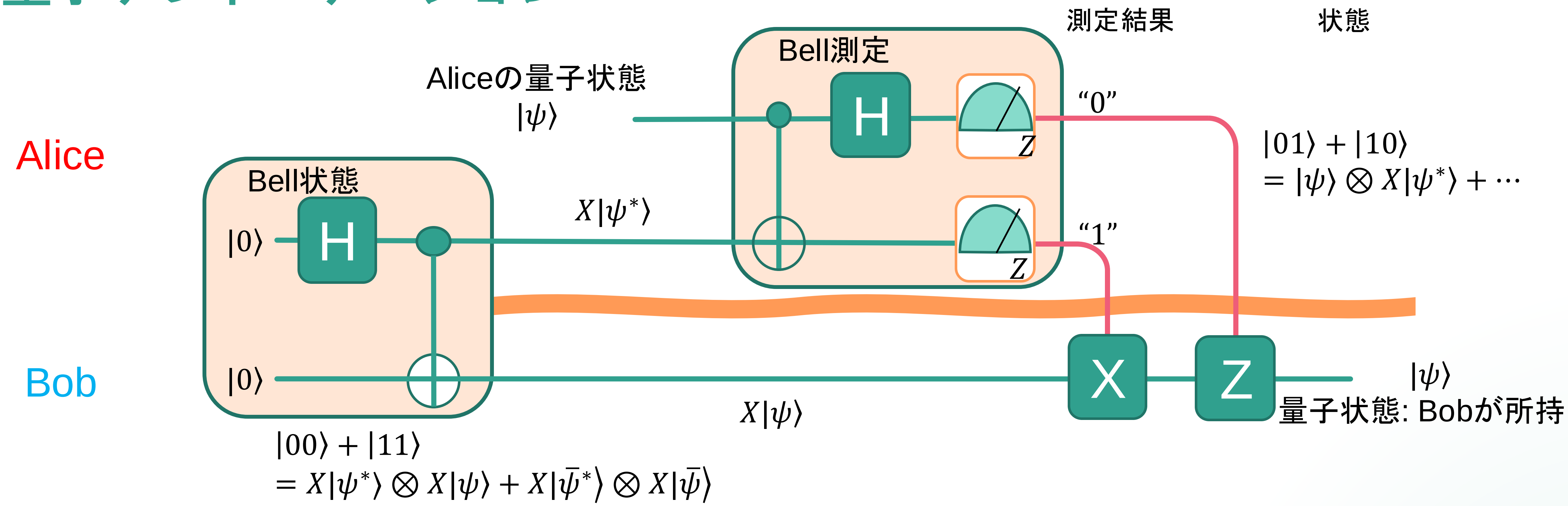

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Kazuhiro Seki, Y.K., Tomoya Hayata, Seiji Yunoki, arxiv:2405.07613

Current Status of Quantum Computing

Quantum advantage for artificial tasks (RCS)

Google '19, '24;
Zuchongzhi '21;
Quantinuum '24

Quantum dynamics?
(what else?)

Fault-tolerant QC

- Prime factoring
- QFT simulation
- Many more…

time

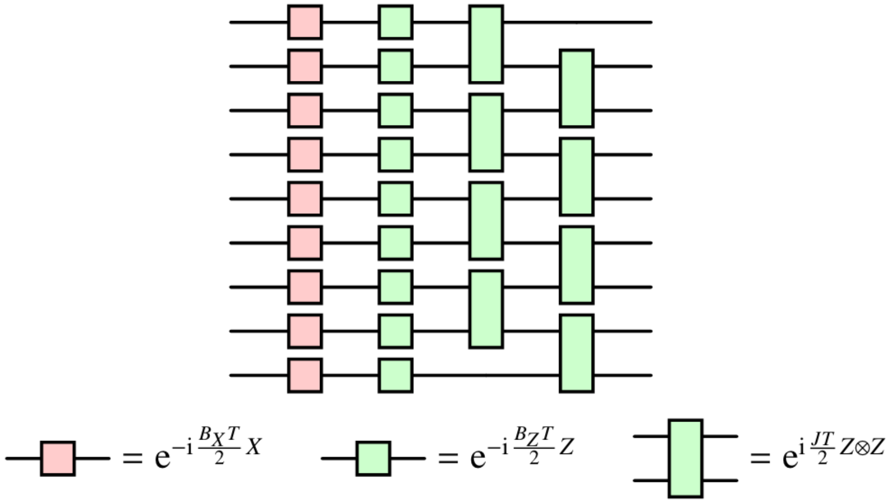
Need to cope with significant noise!!

(What can we do with **50-100 qubits** and **gate fidelity** $\mathbf{99.9\%}$, $\mathbf{99.99\%}$, … ?)

Information scrambling by Floquet circuits

Kicked-Ising model
Prosen '02, '07

- $U_{\mathrm{F}} = \mathrm{e}^{-\mathrm{i}H_Z\frac{T}{2}}\mathrm{e}^{-\mathrm{i}H_X\frac{T}{2}},$
- $H_Z = -J\sum_i Z_i Z_{i+1} + B_Z \sum_i Z_i, \quad H_X = B_X \sum_i X_i$
- Maximally chaotic point at $|JT| = |B_X T| = \frac{\pi}{2}$



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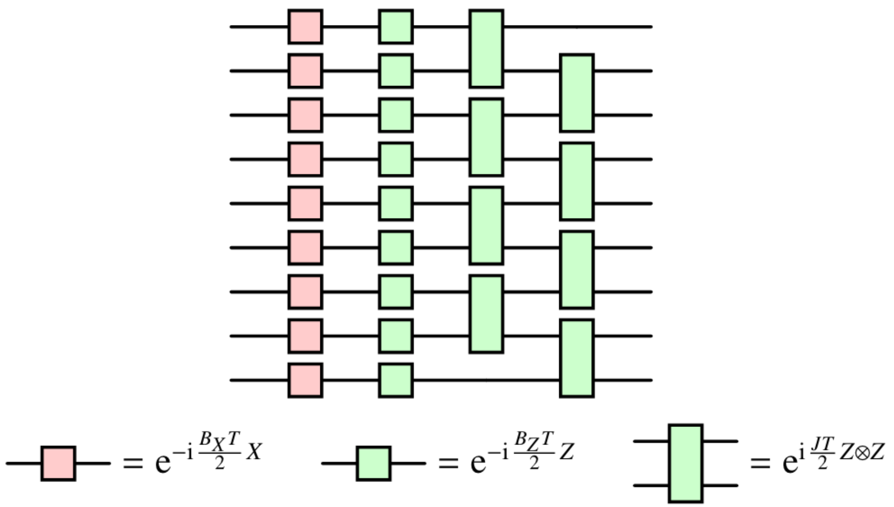


Information scrambling:

- A process of the lost information spreading across the system
- Scrambling dynamics makes it hard to recover the initial information

→ Diagnose the complexity of dynamics

Experiments on trapped-ion quantum computers

GOAL:

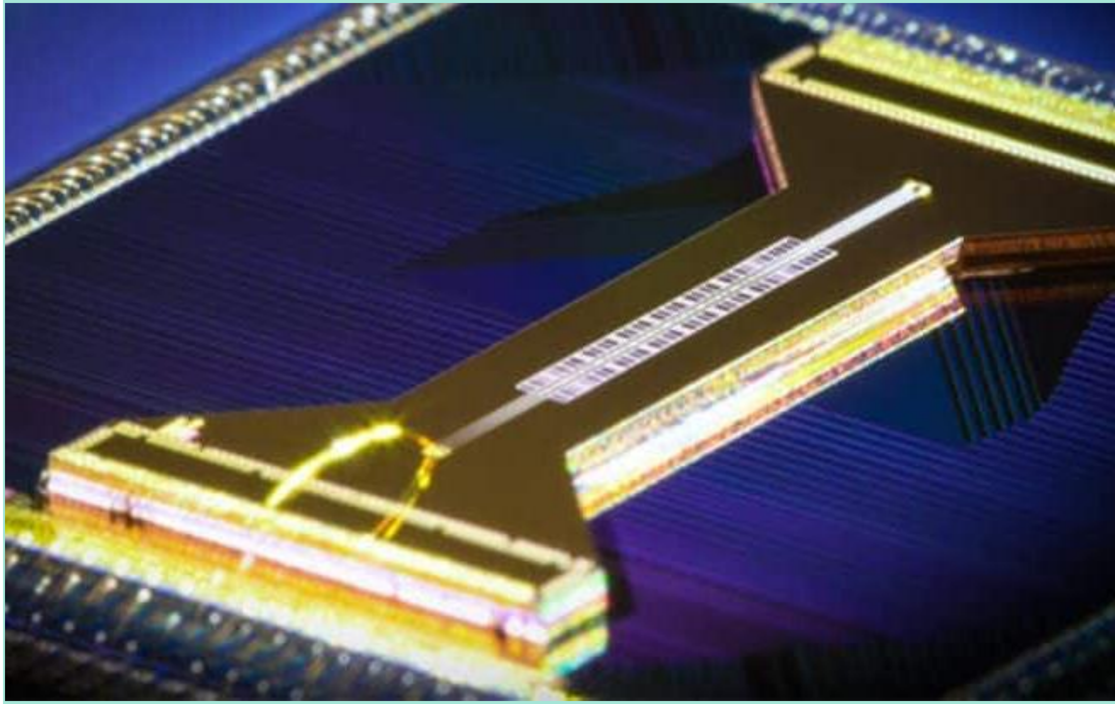
Access the feasibility of scrambling simulation on the current hardware

Seki, YK, Hayata, Yunoki '24

- ❑ Hayden-Preskill recovery protocol
- ❑ Interferometric protocol for OTOCs
- ❑ Thermal expectation value with microcanonical TPQ states

System Model H1

Available on various platforms

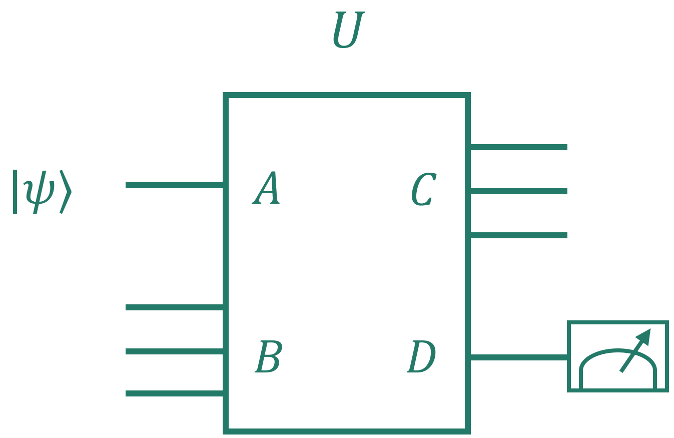


- 99.91% Two-qubit gate fidelity (arbitrary angle)
- 20 qubits
- 99.998% Single-qubit gate fidelity
- Measurement cross talk error < 0.01%
- All-to-all-connectivity
- SPAM fidelity > 99.7%

Hayden-Preskill recovery (HPR) protocol [Theory]

Information scrambling

Hayden & Preskill '07;
Sekino & Susskind '08

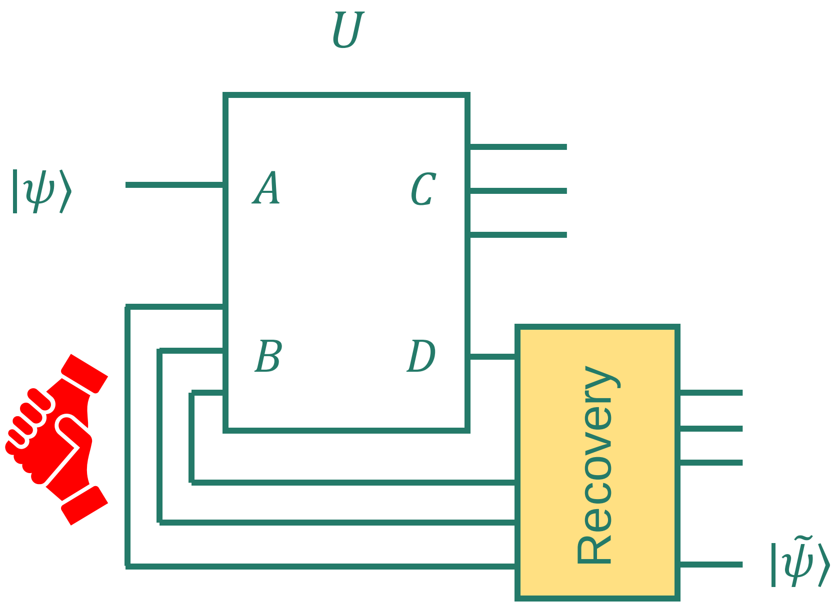


- Cannot learn about $|\psi\rangle$ from any local measurement if U scrambles information

Hayden-Preskill recovery (HPR) protocol [Theory]

Information recovery

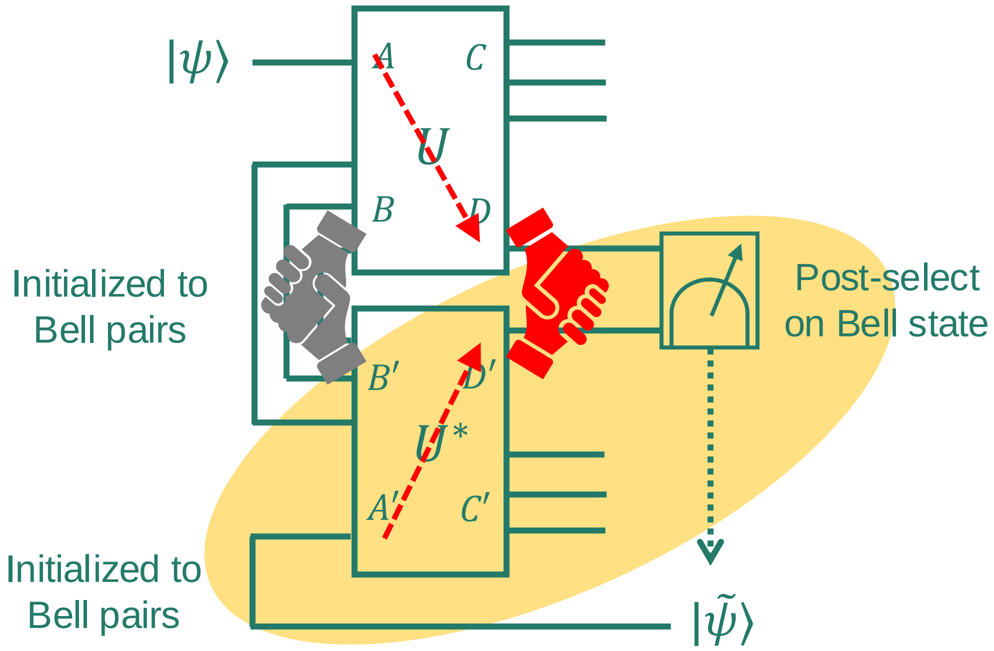
Hayden & Preskill '07



- Entanglement resource helps the recovery of input state.
- $F_{\mathrm{EPR}} = |\langle\psi|\tilde{\psi}\rangle| \approx 1$ at late times if U is scrambling
- What is the recovery protocol?

Hayden-Preskill recovery (HPR) protocol [Theory]

Recovery protocol   Yoshida & Kitaev '17



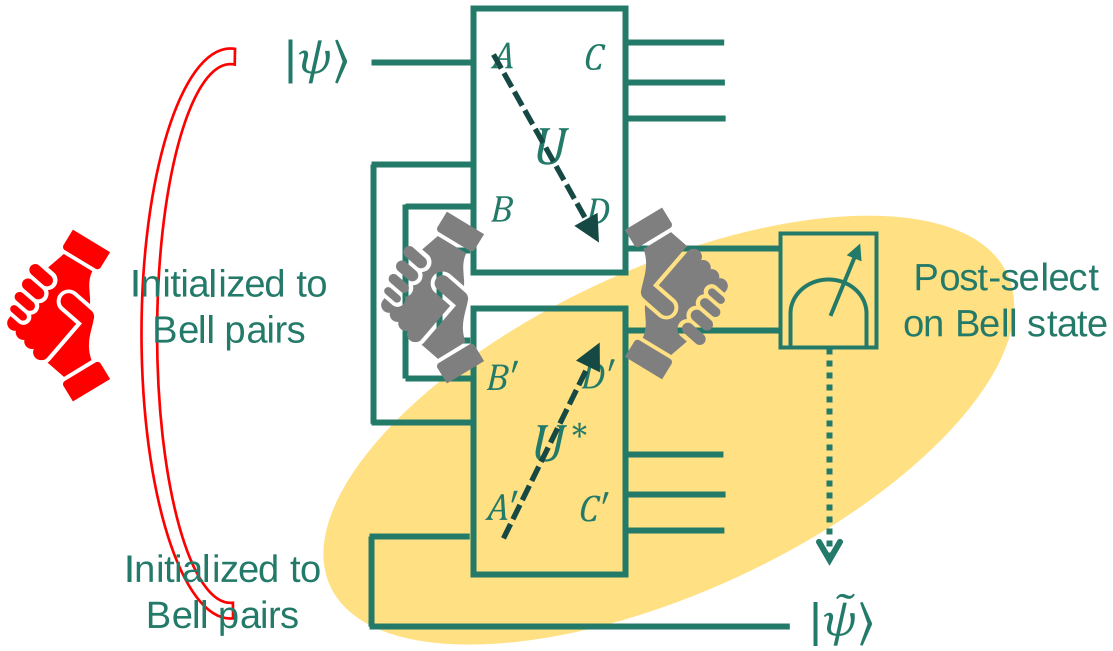
1. States on B and B' are maximally entangled
2. Information propagates from A to D
3. Information propagates from A' to D'
4. States on D and D' are projected on a Bell state

   → States on A and A' are maximally entangled

Hayden-Preskill recovery (HPR) protocol [Theory]

Recovery protocol

Yoshida & Kitaev '17



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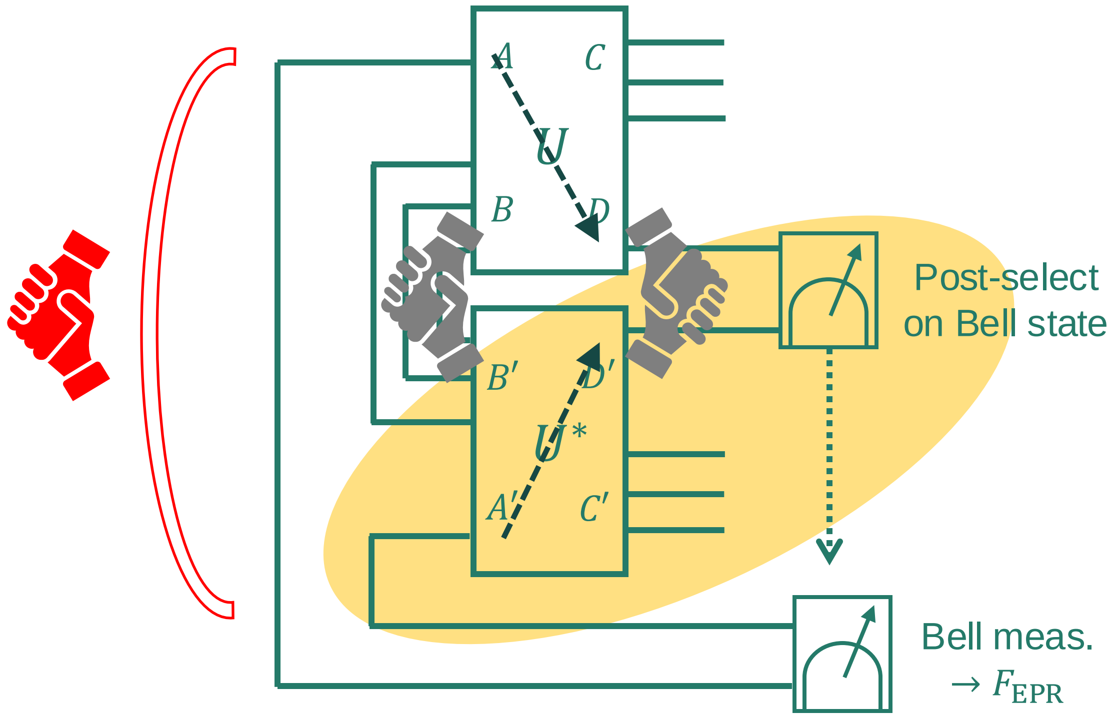

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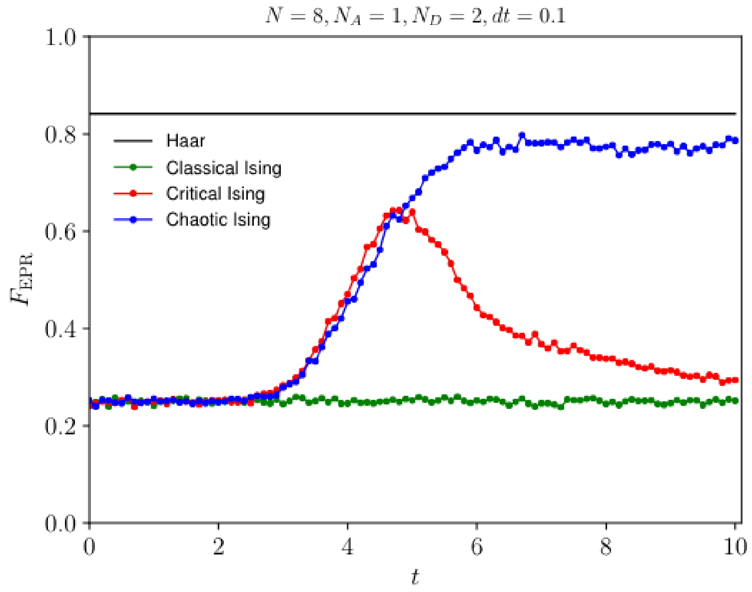


Hayata, Hidaka, YK '21

$$H = -J\sum_i Z_i Z_{i+1} + B_Z \sum_i Z_i + B_X \sum_i X_i$$

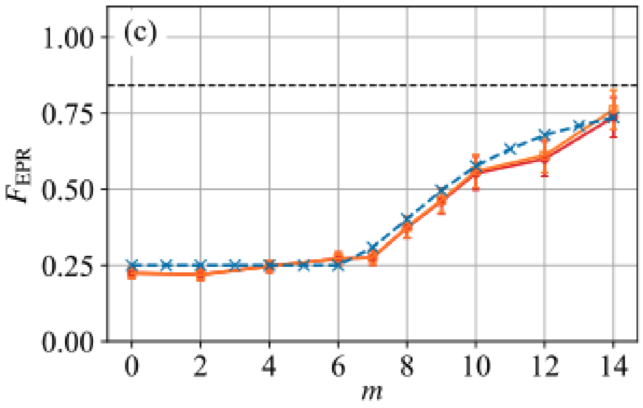
Hayden-Preskill recovery (HPR) protocol [Experiment]

Seki, YK, Hayata, Yunoki ‘24

Setup

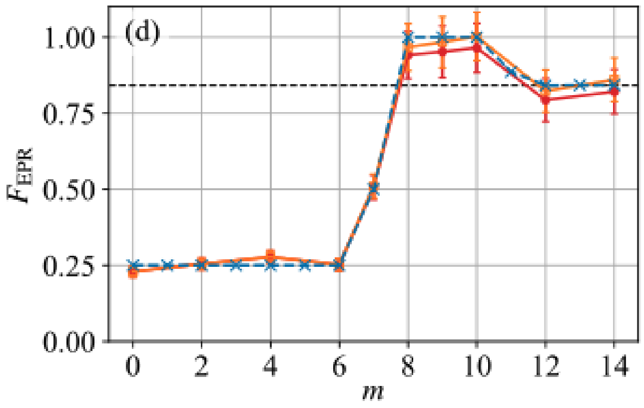
- 2 copies of 9-qubit spin chain + 2 ancilla qubits
- $U = \left(\mathrm{e}^{-\mathrm{i}H_Z\frac{T}{2}}\,\mathrm{e}^{-\mathrm{i}H_X\frac{T}{2}}\right)^m$

$JT = 1.3$



Maximally chaotic

$JT = \frac{\pi}{2}$



dashed: exact, red: raw, orange: mitigated

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- Mitigation is done assuming the global depolarizing channel on U:

$$\rho^{\text{noisy}} = fU^\dagger\rho_{\text{in}}U + (1-f)\frac{I}{2^N}$$

$$f = \left(1 - p_{2\text{Q}}\right)^{N_{2\text{Q}}}$$

$JT = 1.3$

Maximally chaotic

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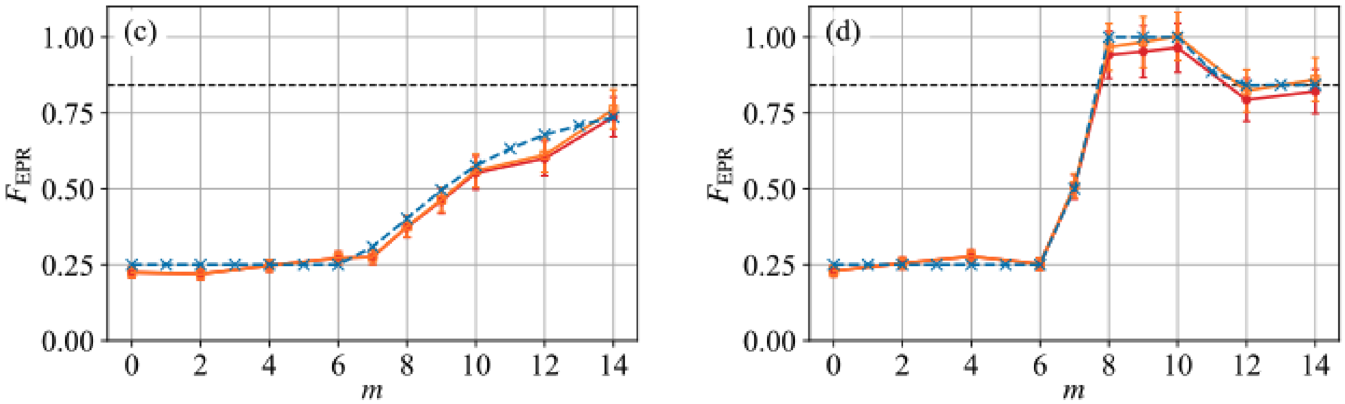

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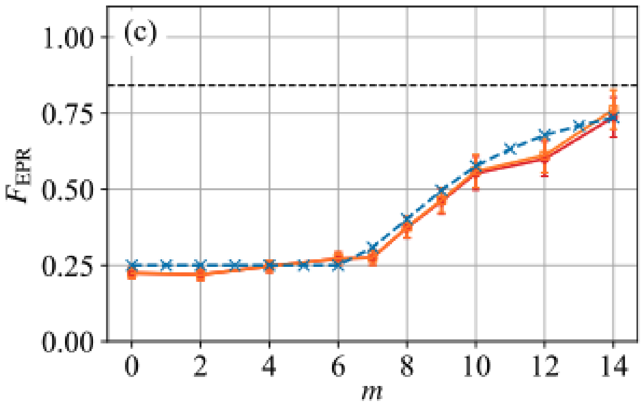

Maximally chaotic

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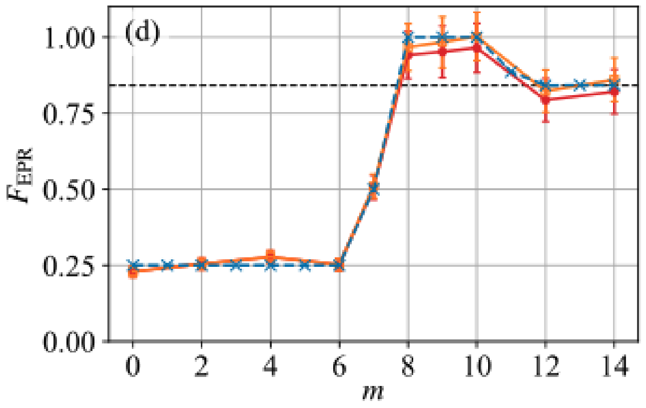
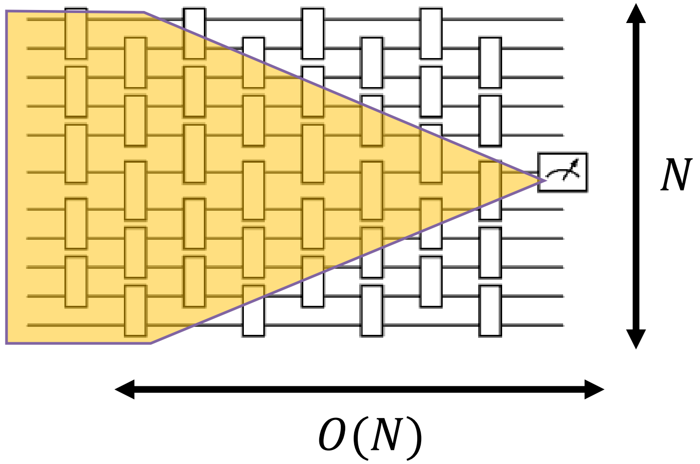


dashed: exact, red: raw, orange: mitigated

Observations

- A sharp increase once the information reaches the measured register in the chaotic model.
- Consistent with Haar random value (dashed lines) at late times.
- Depolarizing channel models the chaotic model better.

Discussion



Circuit fidelity is crudely (sometimes accurately) estimated by

$$f = (1 - p_{2Q})^{N_{2Q}}$$

Geometrically local models requires $\mathrm{poly}(N)$ gates to scramble

- 1D geometrically circuit requires $t \approx O(N)$ for the entire system to get involved: $N_{\mathrm{gate}} \sim O(N^2)$
- E.g. we used 200 2Q gates for the 20-qubit 1D system
  $\Longrightarrow$ 2 copies of 30-qubit system requires 9 times larger gate counts ~ 1800 2Q gates : $f = 0.999^{1800} \approx 0.17$

Discussion

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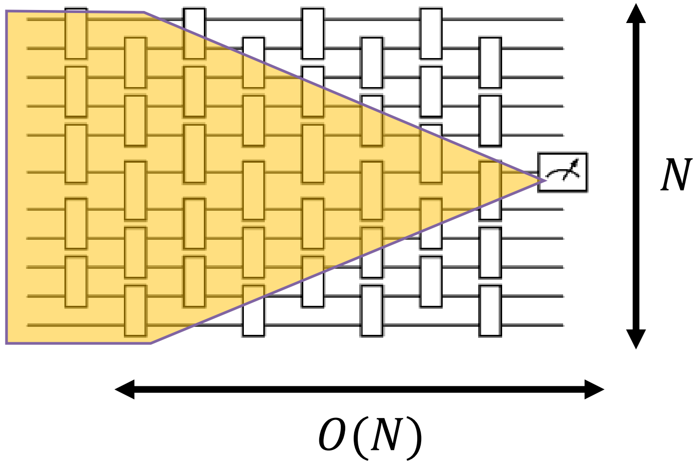
$$f = (1 - p_{2Q})^{N_{2Q}}$$



Geometrically local models requires $\mathrm{poly}(N)$ gates to scramble

- 1D geometrically circuit requires $t \approx O(N)$ for the entire system to get involved: $N_{\mathrm{gate}} \sim O(N^2)$
- E.g. we used 200 2Q gates for 2 copies of 10-qubit 1D systems

  $\Longrightarrow$ 2 copies of 30-qubit system requires 9 times larger gate counts ~ 1800 2Q gates: $f = 0.999^{1800} \approx 0.17$

What if hardware result deviates from the estimate?

- Memory error (on idling qubits, during ion transport)
- SPAM error (bias between $0 \to 1$ and $1 \to 0$)
- Gate counting analysis overestimates the error

Schiffer, Rubio Trivedi & Cirac '24
Granet & Dreyer '24
Chertkov, Chen, Lubasch, Hayes, Foss-Feig '24