

$L_\mu - L_\tau$ 对称性

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Plan to talk

- 1. Introduction
- 2 . Lepton Flavor
 - 2.1 探索
 - 2.2 レプトンフレーバー保存則
- 3. Standard Modelを超える物理
 - 3.1 ニュートリノ振動
 - 3.2 ミューオンの異常磁気能率
 - 3.3 IceCube Gap
 - 3.4 Hubble tension
- 4. $L_\mu - L_\tau$ **ゲージ対称性**
- 5. Summary

1. Introduction

素粒子物理学が目指すもの

この世界(宇宙)を構成する
根本的な存在(素粒子)

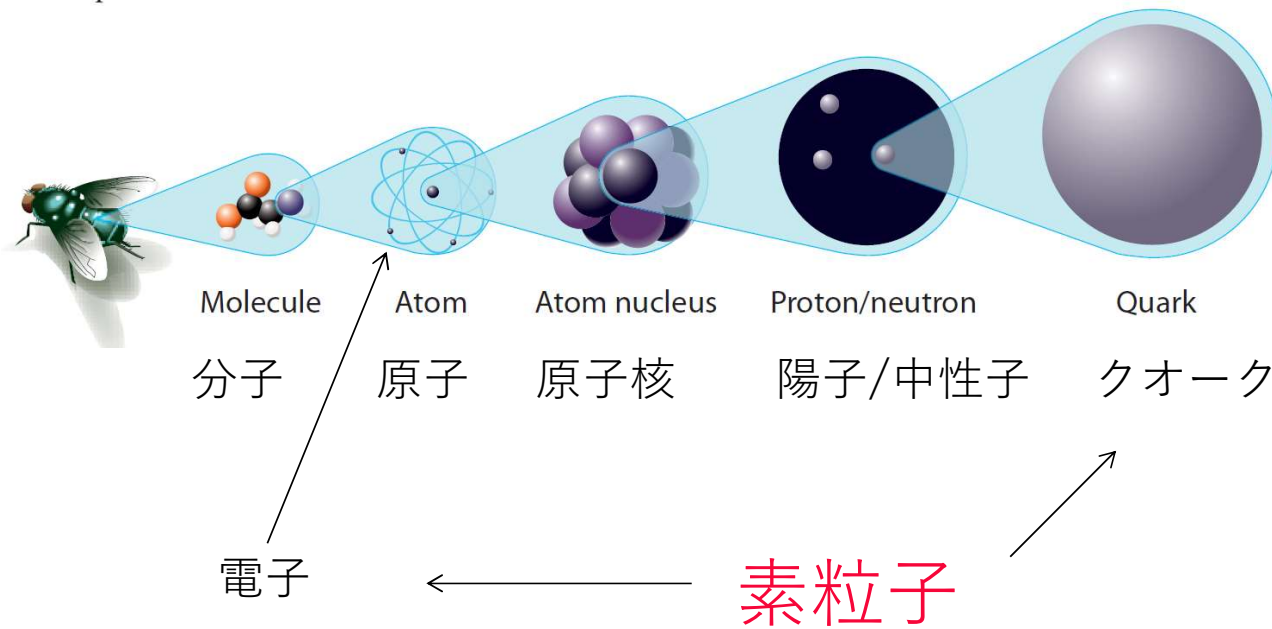
と

それらの間に働く相互作用(力)
を解明すること

素粒子とは？

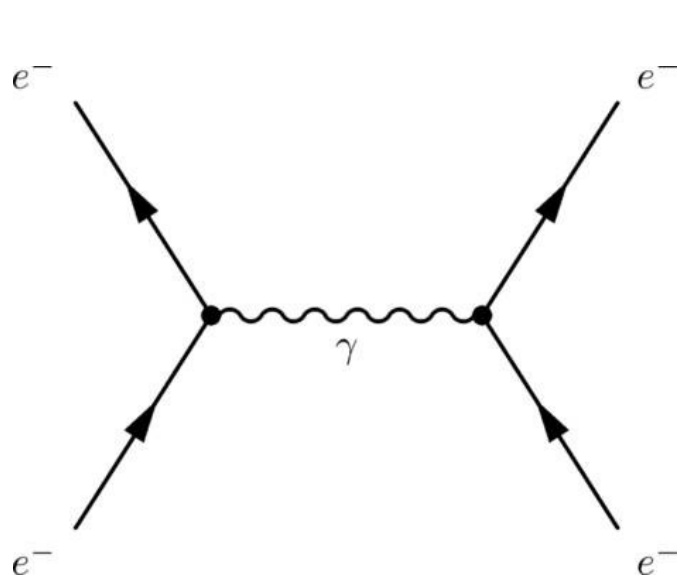
物質を細分化してたどり着く最も小さい存在

http://nobelprize.org/nobel_prizes/physics/laureates/2008/info.pdf

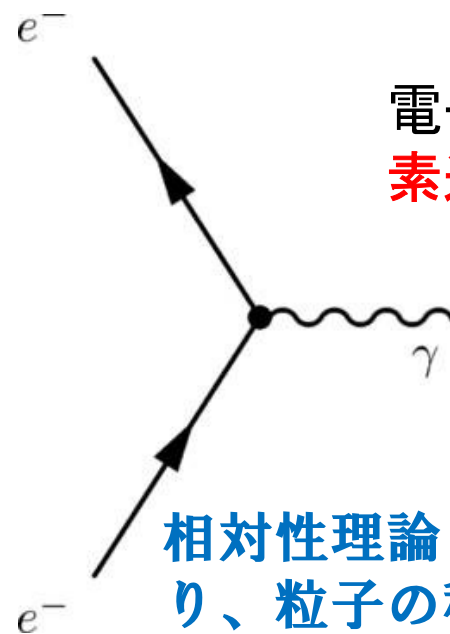


相互作用とは？

素粒子と素粒子の衝突仕方



電子・電子散乱
e: 電子、 γ : 光子(電磁波)



電子と光子の衝突
素過程

相対性理論と量子力学により、粒子の種類を指定すると相互作用は決まる

素粒子物理学

物理学：理論と実験

南部陽一郎(2008年ノーベル賞受賞)によると

素粒子論

湯川秀樹が

1934年にパイ中間子の存在を
理論的に予言したことに始まる物理学の一分野



1949年ノーベル賞

素粒子実験

アーネストローレンスが

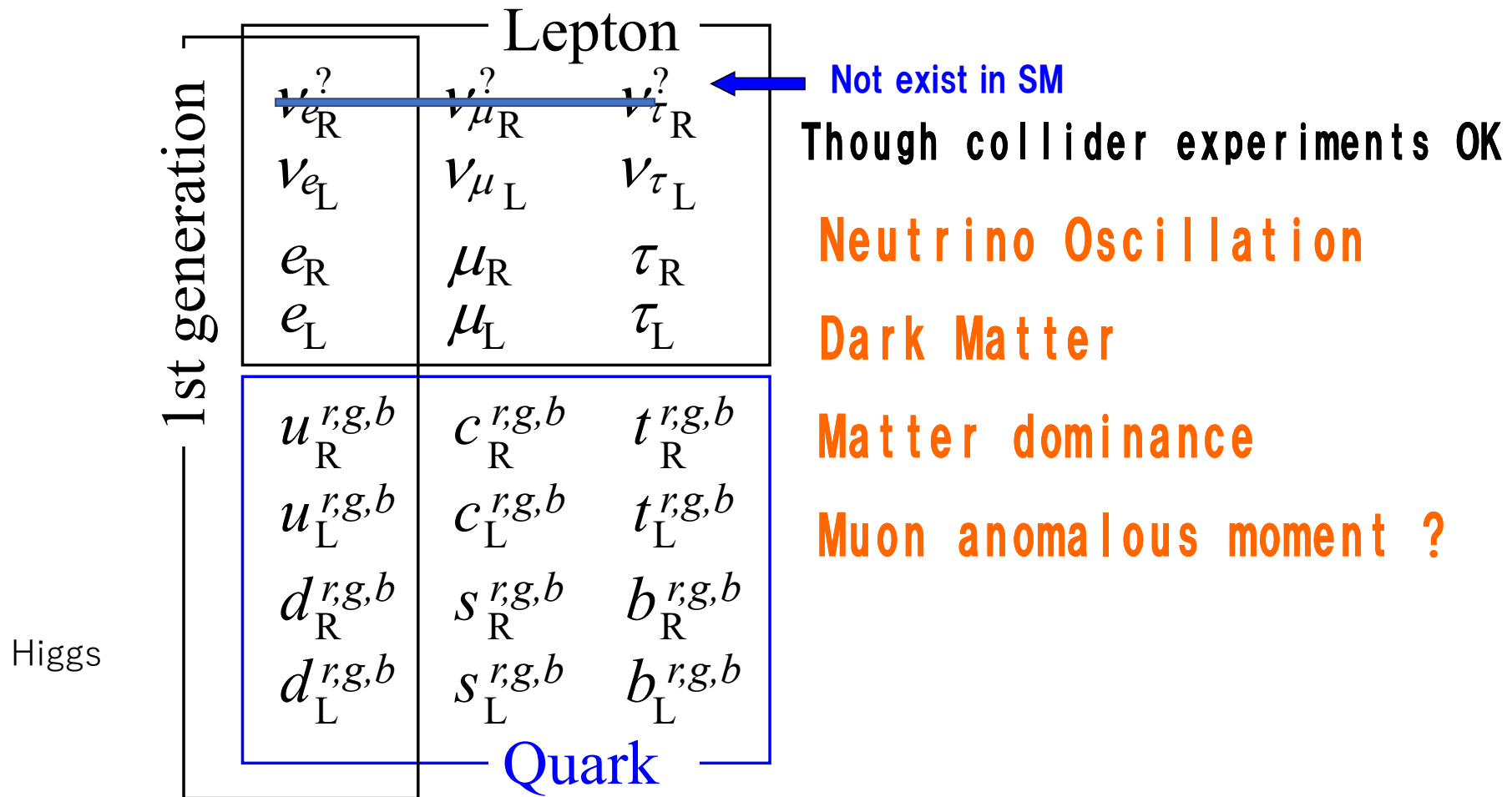
1930年にサイクロトロン(素粒子加速器)の基礎理論を発表



1939年ノーベル賞

Standard Model

$SU(3) \times SU(2) \times U(1)$ gauge group



日本に於ける素粒子物理学 ノーベル賞を例にと

湯川秀樹が1934年にパイ中間子の存在を
理論的に予言したことに始まる物理学の一分野

1949年ノーベル賞

朝永振一郎: 量子電気力学の基礎的研究(1947年)

1965年ノーベル賞

小柴昌俊: ニュートリノの検出(超新星1987A、太陽ニュートリノなど)

2002年ノーベル賞

小林誠・益川敏英: 小林益川理論(CP対称性の破れと6種類のクォーク)

南部陽一郎: 自発的対称性の破れ

2008年ノーベル賞

梶田隆章: ニュートリノ振動

2015年ノーベル賞

詳しく知りたい人は横浜国立大学素粒子論研究室へどうぞ

理論

現象を説明するための仮設・模型（モデル）
より少ない仮定でより多くのことを説明したい

実験による検証

どの理論が「よい」理論か。そもそも間違っていないか
未知の現象はないか

2. レプトンフレーバー

ニュートリノが0質量であることから自動的に出てくる保存「電荷」

電子数、ミューオン数、タウ数

$$L_e \quad L_\mu \quad L_\tau$$

$$\begin{matrix} e^- & \nu_e & \mu^- & \nu_\mu & \tau^- & \nu_\tau \end{matrix}$$

$$L_e \quad 1 \quad 1$$

反粒子は - 1

$$L_\mu \quad \quad \quad 1 \quad 1$$

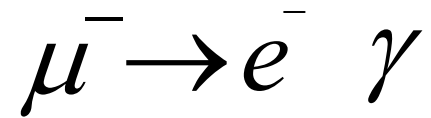
$$L_\tau \quad \quad \quad \quad \quad 1 \quad 1$$

例

$$L_\mu \quad \pi^- \rightarrow \mu^- \quad \bar{\nu}_\mu$$
$$0 = 1 + (-1)$$

2. 1 荷電レプトンによるレプトンフレーバー破れの探索

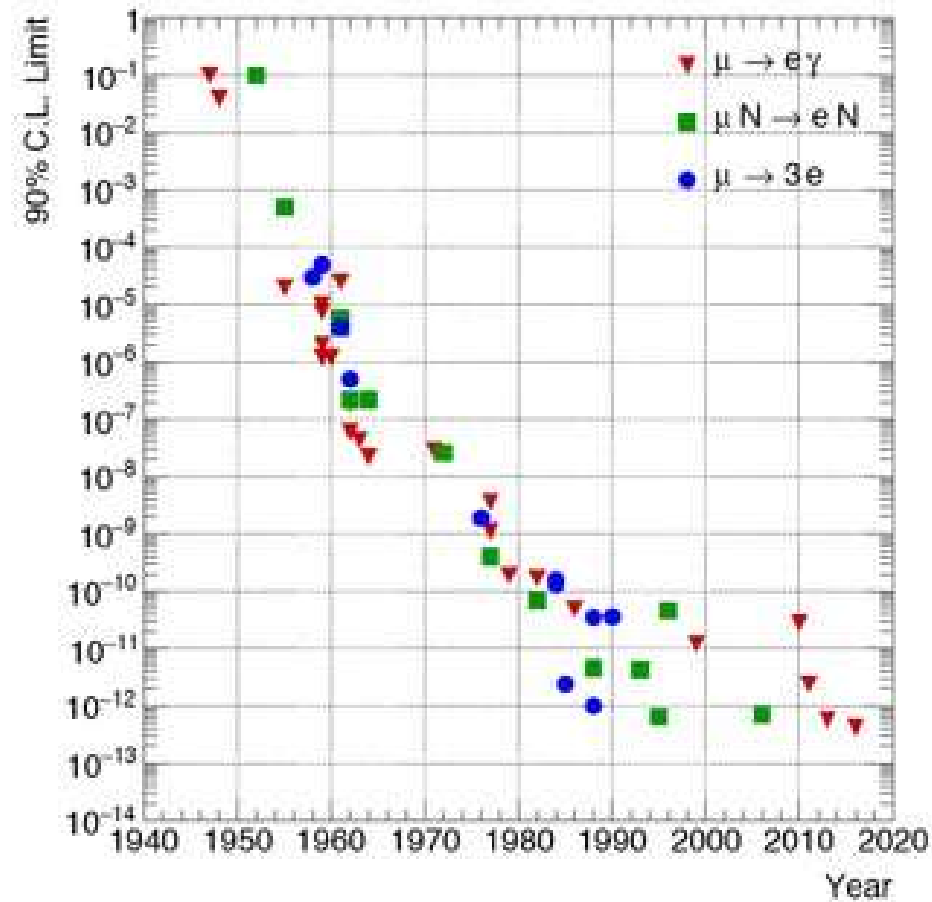
ニュートリノに質量があると、一般にはレプトンフレーバーは保存しないので、



$$L_\mu \quad 1 = 0 + 0$$

$$L_e \quad 0 = 1 + 0$$

が起こりうる。



LFV from muon decay

	Upper limit on Br
$\mu^+ \rightarrow e^+ \gamma$	$< 5.7 \times 10^{-13}$
$\mu^+ \rightarrow e^+ e^+ e^-$	$< 1.0 \times 10^{-12}$
$\mu^- \text{Ti} \rightarrow e^- \text{Ti}$	$< 6.1 \times 10^{-13}$
$\mu^- \text{Au} \rightarrow e^- \text{Au}$	$< 7 \times 10^{-13}$

MEG experiment updates/discovers(?)

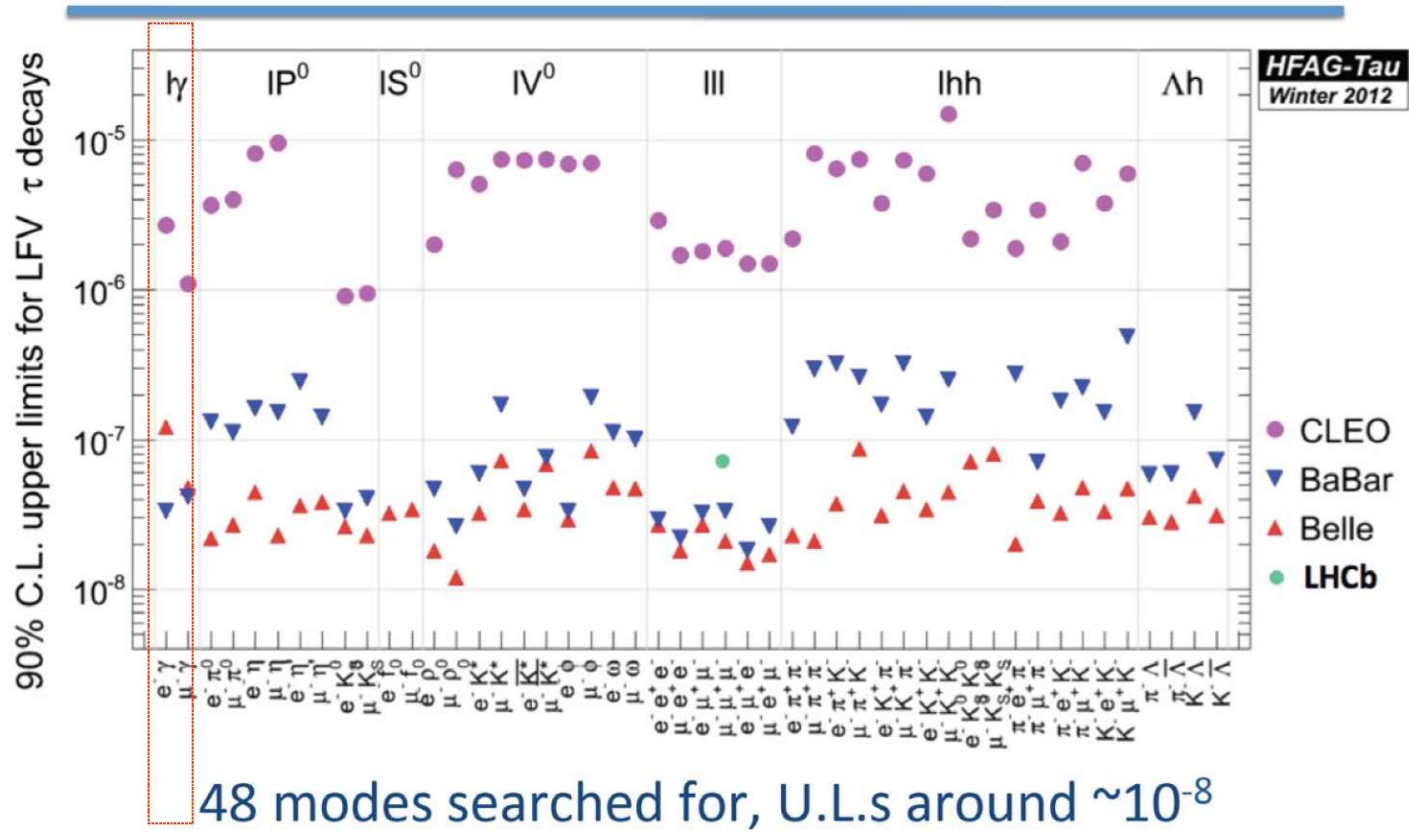


COMET/Mu2E will discover(?)



In near future

τ の崩壊



2.2 Lepton flavor保存の導出：ネーターの定理

Lepton Part Only

$$L_i = \begin{pmatrix} \nu_{Li} \\ e_{Li} \end{pmatrix}, \quad e_{Ri}, \quad (i = 1, 2, 3)$$

Kinetic Part

$$\mathcal{L}_k = \bar{L}_i i \not{D}_L L_i + \bar{e}_{Ri} i \not{D}_R e_{Ri}$$

$$D_{L\mu} = \begin{pmatrix} \partial_\mu + \frac{i}{2}g_1 B_\mu - g_2 \frac{i}{2} W_\mu^0, & g_2 \frac{i}{\sqrt{2}} W_\mu^+ \\ g_2 \frac{i}{\sqrt{2}} W_\mu^-, & \partial_\mu + \frac{i}{2}g_1 B_\mu + \frac{i}{2}g_2 W_\mu^0 \end{pmatrix}$$

$$D_{R\mu} = \partial_\mu + ig_1 B_\mu$$

$$\mathcal{L}_k = \mathcal{L}_{k,diag} + \mathcal{L}_{k,W}$$

$$\mathcal{L}_{k,diag} = \bar{\Phi}_j i \not{D} \Phi_j \quad \Phi_j = \{ \nu_{Li}, e_{Li}, e_{Ri} \} \quad \text{「内積」の形}$$

$$D_\mu = \partial_\mu - ieQ A_\mu - ig_Z (T_3 - Q \sin^2 \theta_w) Z_\mu$$

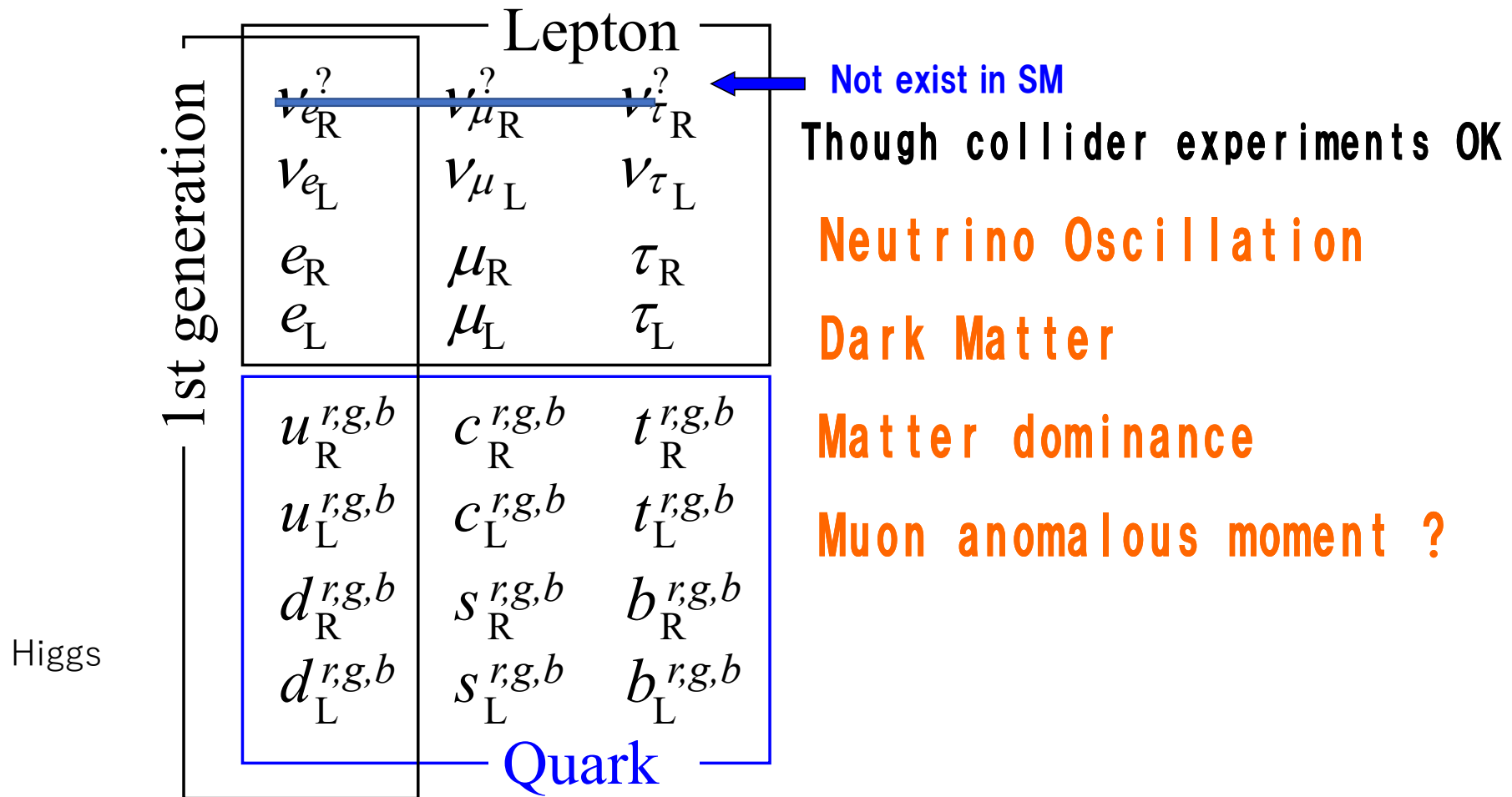
$$U_l, \quad l = \nu_L, e_L, e_R, \quad 3 \times 3 \text{ Unitary Matrix}$$

$$l \rightarrow U_{il} (e_{Li} \rightarrow (U_{eL} e_L)_{i\text{よ}})$$

と、基底を変えても同じ形の式が成立 U_l は互いに違うもので良い

Standard Model

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と、基底を変えても同じ形の式が成立 U_l は互いに違うもので良い

$$\mathcal{L}_{k,W} = ig_2 \frac{1}{\sqrt{2}} W_\mu^+ \bar{\nu}_{Li} \gamma_\mu e_{Li} + h.c.$$

こちらを不変に保つには $U_{\nu L} = U_{eL}$ が必要
Higgs Part

$$\mathcal{L}_H = Y_{ij} \bar{L}_i e_{Rj} + h.c.$$

Y_{ij} 3×3 複素行列 :: 二つのユニタリ行列で対角化出来る

$$Y_{ij} \longrightarrow Y_{diag} = \text{diag}\{y_e, y_\mu, y_\tau\} = U_L Y_{ij} U_R^\dagger$$

$$L_\alpha \equiv U_{L\alpha i} L_i = \begin{pmatrix} U_{L\alpha i} \nu_{Li} \\ U_{L\alpha i} e_{Li} \end{pmatrix}, \quad e_{R\alpha} \equiv U_{Ri} e_{Ri}, \quad \alpha = e, \mu, \tau$$

$$\begin{aligned} \longrightarrow \mathcal{L}_H &= Y_\alpha \bar{L}_\alpha e_{R\alpha} + h.c. \\ &= h^+ (y_e \bar{\nu}_{eL} e_{eR} + y_\mu \bar{\nu}_{\mu L} \mu_{eR} + y_\tau \bar{\nu}_{\tau L} \tau_{eR}) \\ &\quad + h^0 (y_e \bar{e}_{eL} e_{eR} + y_\mu \bar{\mu}_{eL} \mu_{eR} + y_\tau \bar{\tau}_{eL} \tau_{eR}) + h.c. \end{aligned}$$

運動項も $U_{\nu L} = U_{eL}$ が成立しているので不変

$$\mathcal{L}_{k,diag} = \bar{\Phi}_\alpha i \not{D} \Phi_\alpha \quad \Phi_\alpha = \{\nu_{\alpha L}, e_{\alpha L}, e_{\alpha R}\}$$

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フレーバー基底で「対角化」された形

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フレーバー基底で「対角化」された形

残った対称性 : : レプトンフレーバー

$$\Phi_\alpha = \{\nu_{L\alpha}, e_{L\alpha}, e_{R\alpha}\} \quad \alpha = e, \mu, \tau$$

同じフレーバーを持つものが対で現れる。

→あるフレーバーのみの位相を変換してもラグランジアンは不変

→そのフレーバーの個数が保存する。

$$\{e'_L, e'_R, \nu'_{eL}\} = \exp\{-i\theta_e\} \{e_L, e_R, \nu_{eL}\} \quad \text{と変換}$$

$$\begin{aligned} \mathcal{L}'_{k,W} &= ig_2 \frac{1}{\sqrt{2}} W_\mu^+ \bar{\nu}'_e \gamma_\mu e'_L + h.c. \\ &= ig_2 \frac{1}{\sqrt{2}} W_\mu^+ \bar{\nu}_e e^{i\theta} \gamma_\mu e^{-i\theta} e_L + h.c. = \mathcal{L}_{k,W} \quad \text{など} \end{aligned}$$

ネーターの定理より保存「電流」の存在が分かる。

それぞれのフレーバーに対し

$$j_\alpha^\mu = \bar{\nu}_{L\alpha} \gamma^\mu \nu_{L\alpha} + \bar{e}_{L\alpha} \gamma^\mu e_{L\alpha} + \bar{e}_{R\alpha} \gamma^\mu e_{R\alpha}$$

第0成分の空間積分は「電荷」を与え、それが保存することを定理は保証する。

$$Q_\alpha = \int d^3x j_\alpha^0$$

たとえば $\alpha = e$ に対して、生成消滅演算子で書くと

$$Q_e = L_e = \int d^3p \sum_{l=\nu_{eL}, e_L, e_R} b_l^\dagger(\mathbf{p})b_l(\mathbf{p}) - d_l^\dagger(\mathbf{p})d_l(\mathbf{p})$$

$b^\dagger b$ number operator for particle

$d^\dagger d$ number operator for anti - particle

電子とニュートリノ $L_e = +1$

陽電子と反電子ニュートリノ $L_e = -1$

\not{L}_μ, L_τ も同様

3. 標準理論を超える物理

- 標準理論

高い精度で確立。18個のパラメタで「全て」の現象を説明

- ただし、完璧ではない ..

ニュートリノ振動 (Neutrino masses and lepton mixing)、暗黒物質、物質優勢、

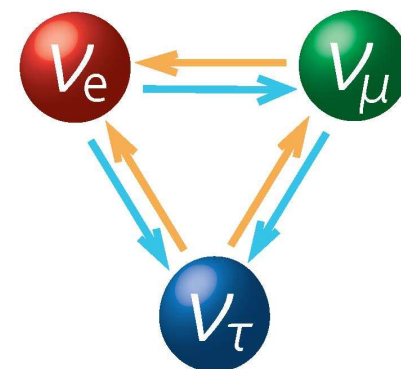
$g_\mu - 2$: ミューオンの異常磁気能率 Hubble-tension IceCube-Gap

- 純理論的にも？

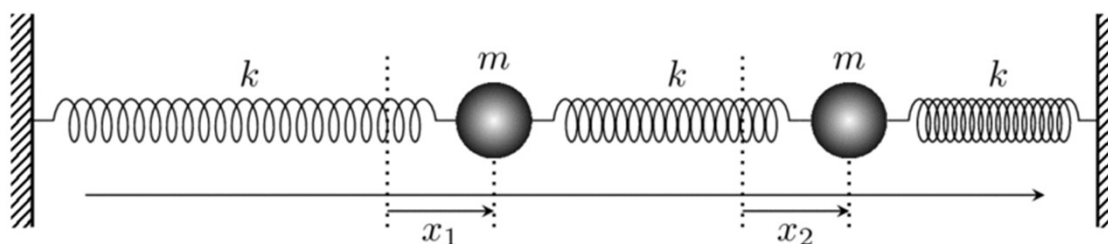
Too many params, Charge quantization, Why three gauges? Why three generations ?

3.1 ニュートリノ振動

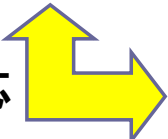
ニュートリノは変身する！！



連成振動との類似性



玉の揺れ幅が入れ替わる

対応  V_e が V_μ と入れ替わる

ニュートリノ / 振動 vs うなり

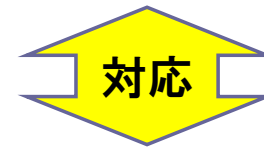
類似の方程式

ニュートリノの遷移振幅

混合の元が必要！！

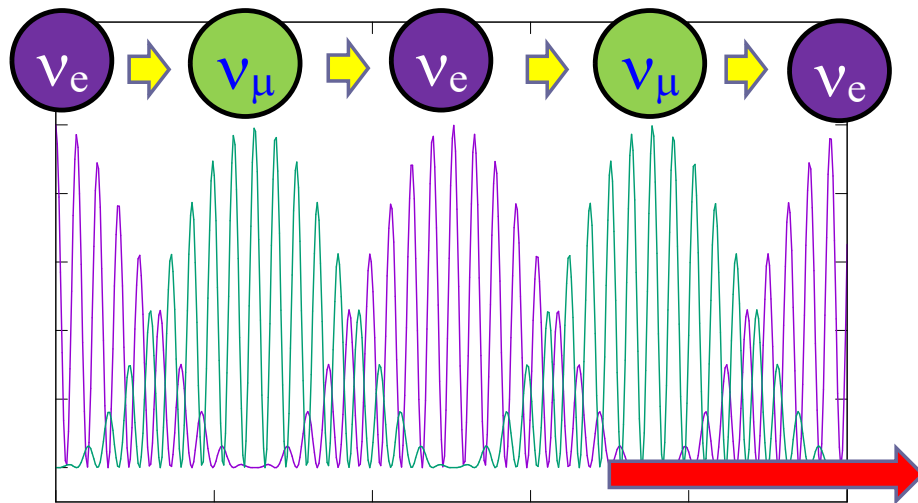
質量が必要！！

$$P_{\nu_\alpha \rightarrow \nu_\beta} = \sin^2 2\theta \sin^2 \left(1.27 \frac{m_i^2 - m_j^2}{4E} L \right)$$



うなりの変位

$$A^2 \sin^2(\omega_{\text{av}} t) \sin^2 \left(\frac{\omega_1 - \omega_2}{2} t \right)$$



飛行距離 (L) によって変化

他の粒子が存在すると、一般には

$$\{\alpha'_L, \alpha'_R, \nu'_{\alpha L}\} = \exp\{-i\theta_\alpha\} \{\alpha_L, \alpha_R, \nu_{\alpha L}\} \quad \alpha = e, \mu, \tau$$

と変換したときにラグランジアンが不変にならない。

→一般にレプトンフレーバーは定義出来ない。

→保存量とならない。

例 ニュートリノ質量（一番簡単な質量の例）、右巻きニュートリノを導入

$$\mathcal{L}_H \quad + = \quad \tilde{H} Y_{\alpha i} \bar{L}_\alpha \nu_{Ri} + h.c.$$

対角化出来ない。∵Lを回転する自由度は残っていないから
→フレーバーの基底で「対角化」できない。

ニュートリノの質量の固有状態は $\mathcal{L}_H = h^0 Y_{\alpha i} \bar{\nu}_{L\alpha} \nu_{Ri}$ を対角化することで得られるので、

$$Y_{\alpha i} \rightarrow U_{MNS}^\dagger Y_{\alpha i} U_{\nu R}^\dagger = \text{diag}\{y_{\nu_e}, y_{\nu_\mu}, y_{\nu_\tau}\}$$

に対し、ニュートリノの質量固有状態は $\nu_{Li} = U_{MNS}^\dagger \nu_\alpha$ でえられる。

$$W_\mu^+ \bar{\nu}_{\alpha L} \gamma_\mu e_{\alpha L} = W_\mu^+ \bar{\nu}_{Li} U_{MNS}^\dagger \gamma_\mu e_{\alpha L}$$

「対角的」でなくなる。→ニュートリノ振動の種

ニュートリノ振動

ニュートリノに質量を与える機構が必要

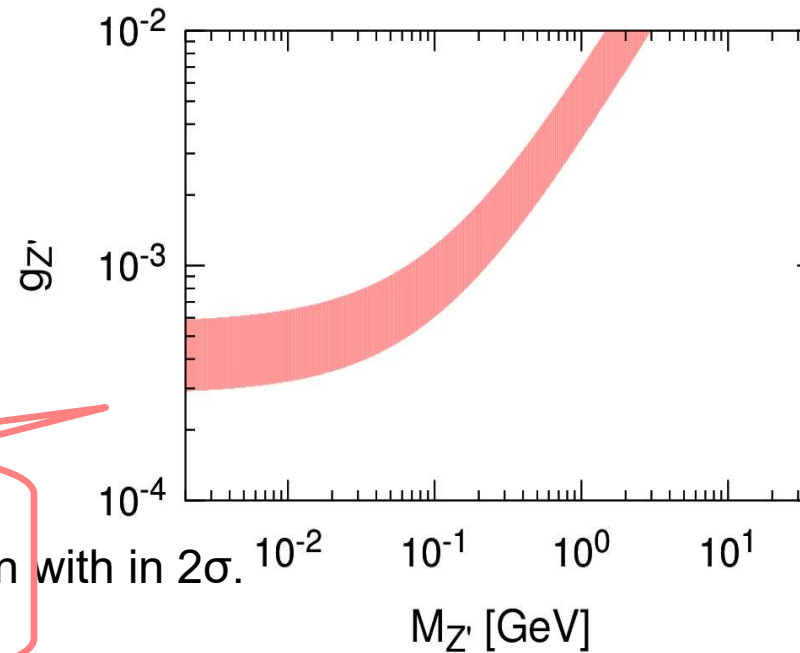
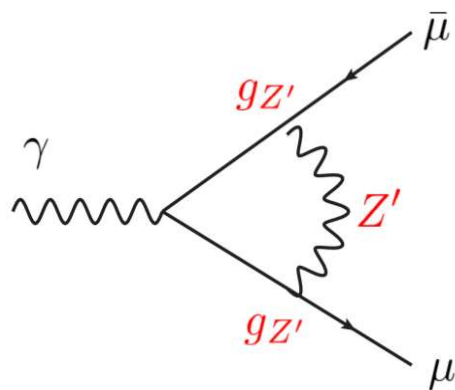
3.2 Muon g-2 discrepancy motivation to muonic force

Hagiwara, Liao, Martin, Nomura, Teubner, JPG38, 085003 (2011)

- The discrepancy between theory(SM) and experiment is

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (26.1 \pm 8.0) \cdot 10^{-10} (3.3\sigma)$$

- In $L_\mu-L_\tau$ models, we have an additional contribution.



The discrepancy of muon $g-2$ can be explained in the red region with in 2σ .

ニュートリノ振動

ニュートリノに質量を与える機構が必要

ミュオンの異常磁気能率

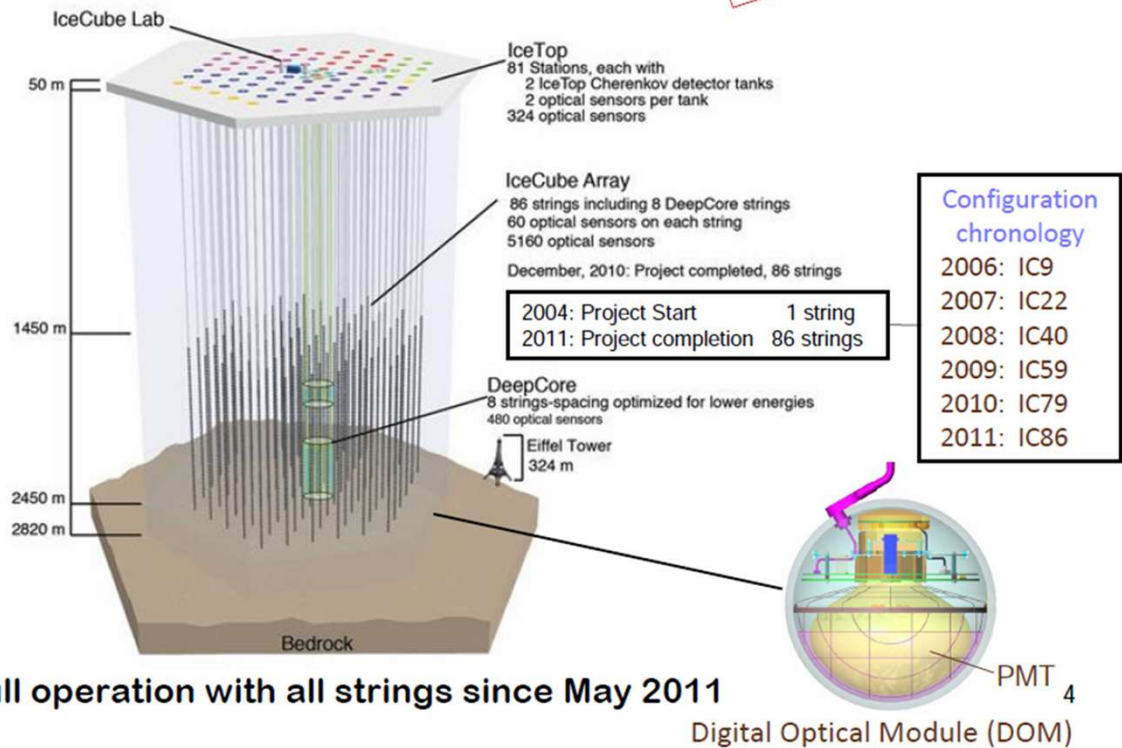
ミュオンの的なものに特異的に働く力が必要

3.3 IceCube Gap

The IceCube Neutrino Observatory

Completed: Dec 2010

Observe
High Energy Neutrinos
Including
Cosmic Neutrino



Full operation with all strings since May 2011

High energy cosmic neutrino

Target: Neutrinos produced in cosmic-ray interactions with gas (p) or radiation (γ), followed by pion decays.

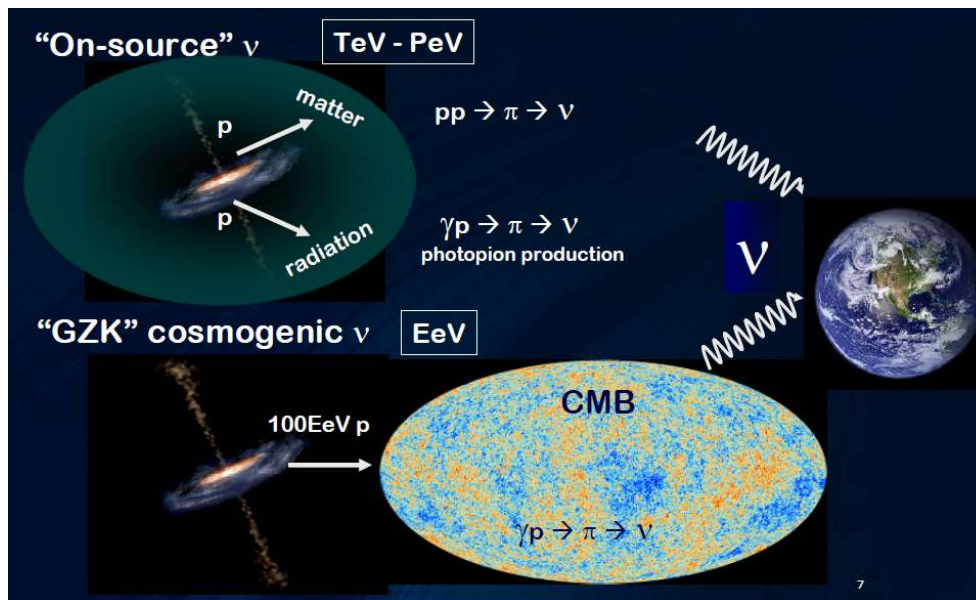
@ Source

$$\nu_e : \nu_\mu : \nu_\tau = 1 : 2 : 0$$

oscillation

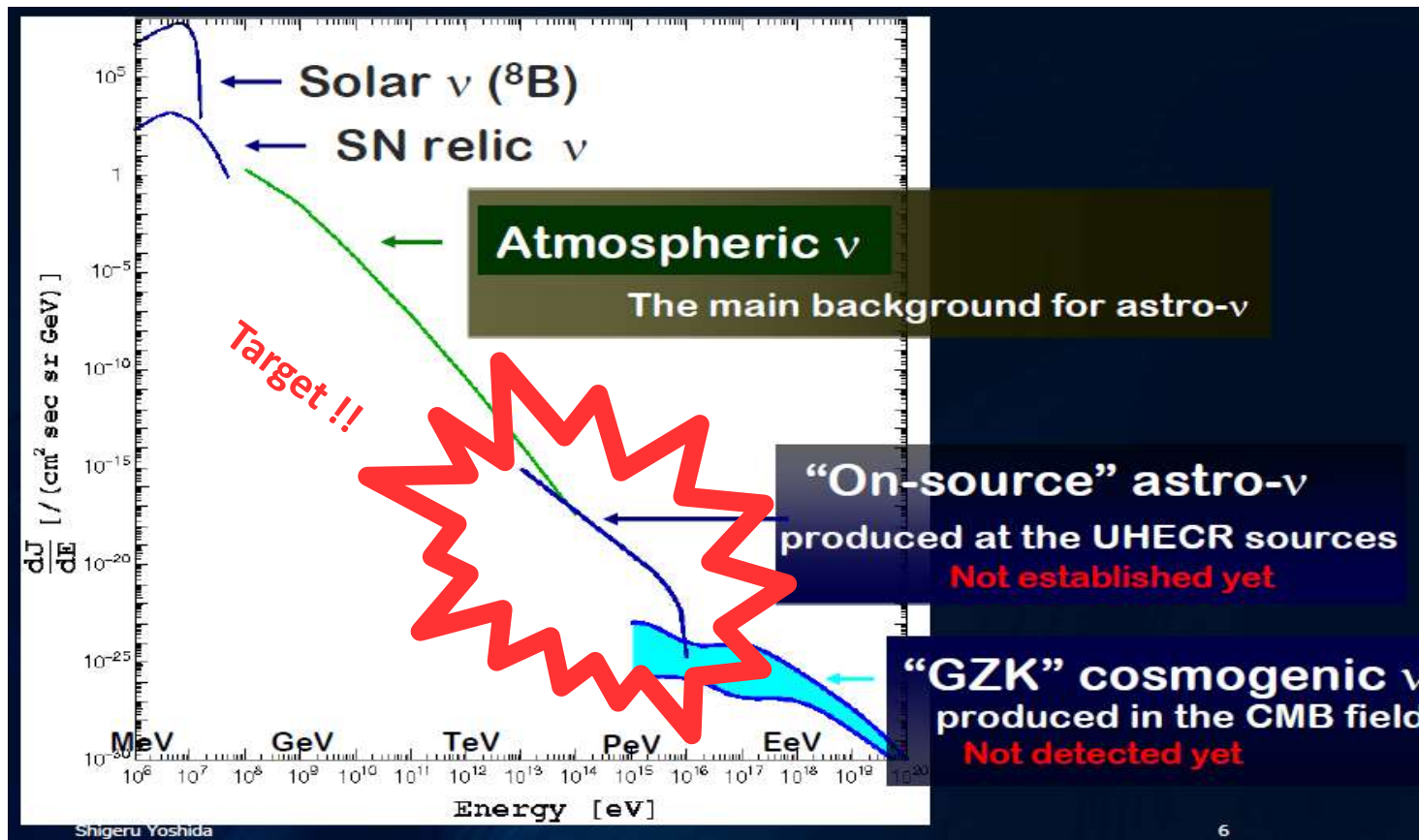
@ Earth

$$\simeq 1 : 1 : 1$$



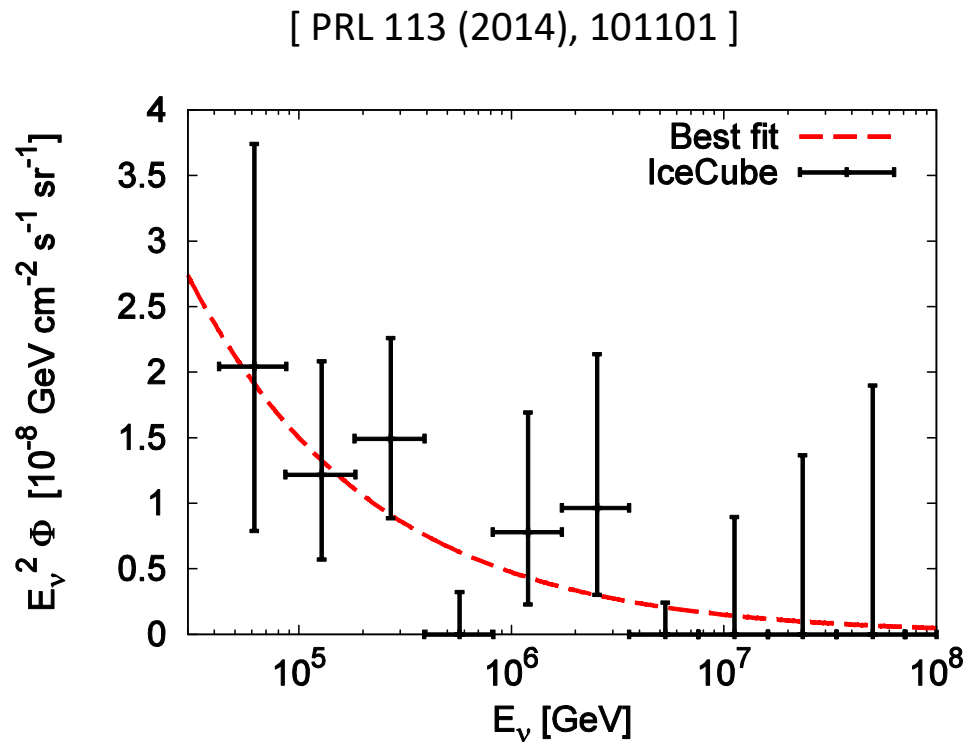
High energy cosmic neutrino

Target: Neutrinos having energies of $O(\text{TeV} - \text{PeV})$.



Three-year data

Neutrino flux ($\nu + \bar{\nu}$) as a function of its energy.



1. It rejects a purely atmospheric explanation at **5.7** sigma.
2. The data are consistent with equal **(1:1:1)** flavor ratios and **isotropic** arrival directions.
3. The best-fit power law is

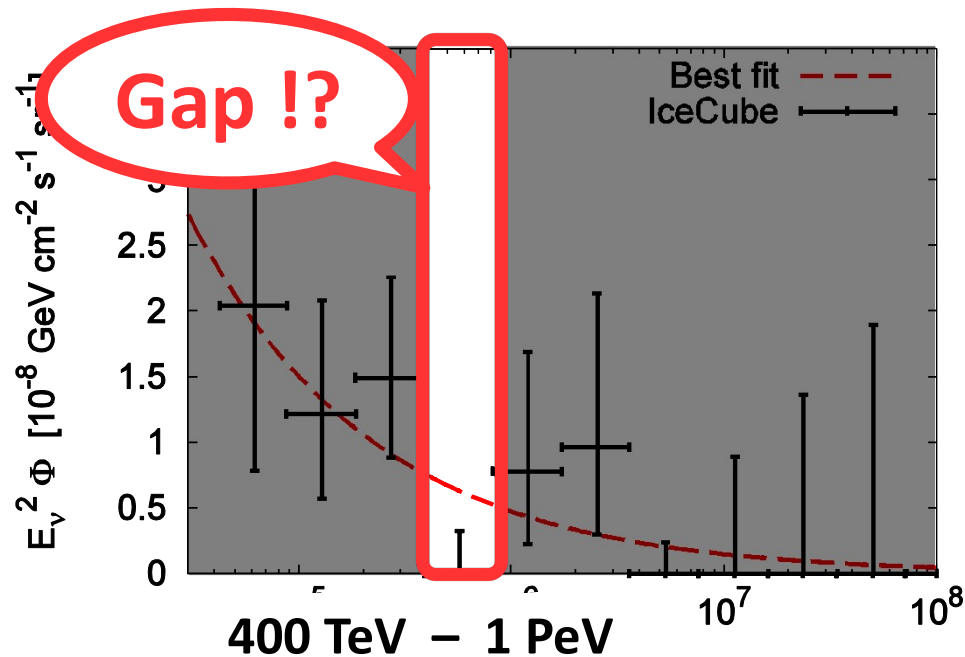
$$\Phi(E) = \phi \left[\frac{E_\nu}{100 \text{ TeV}} \right]^{-2.5}$$

※ Combined analysis:
[Astrophys. J. 809 (2015) 1, 98]

Gap or fluctuation?

Neutrino flux ($\nu + \bar{\nu}$) a function of its energy.

[PRL 113 (2014), 101101]



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ニュートリノ振動

ニュートリノに質量を与える機構が必要

ミュオンの異常磁気能率

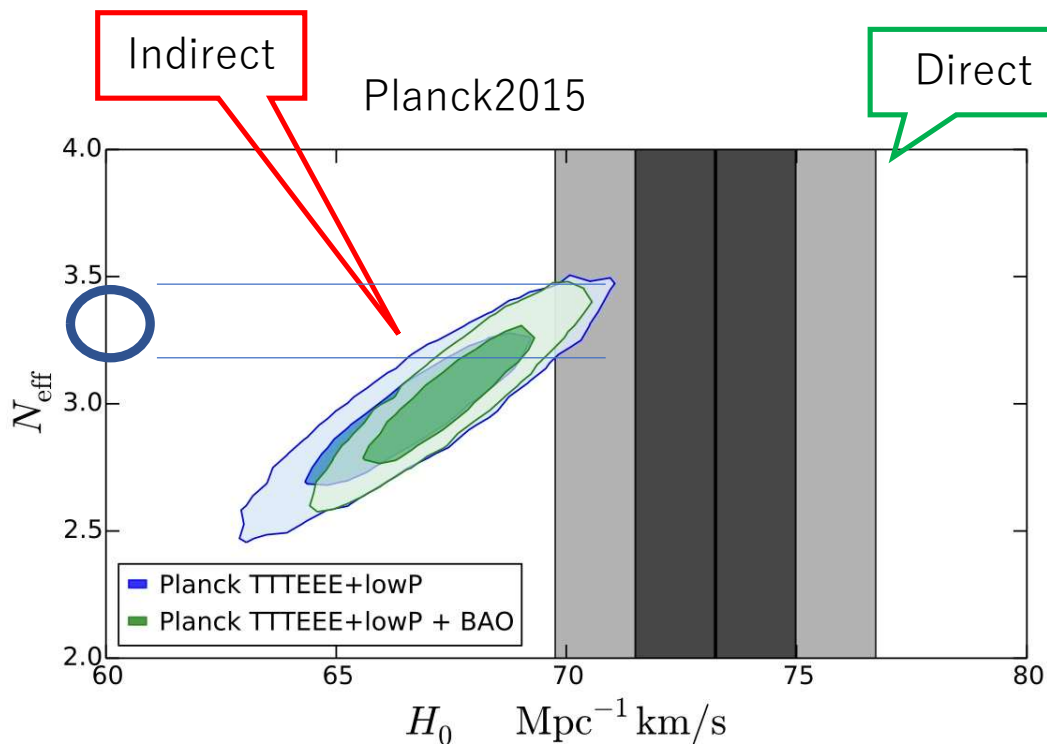
ミュオンのものに特異的に働く力が必要

IceCube Gap

ニュートリノに働く新しい力が必要

3.4 Hubble Tension

Hubble Tension : Tension between two methods to measure H_0



José Luis Bernal, et al. Journal of Cosmology and
Astroparticle Physics, Vol. 2016, No. 10, p. 019 (2016)

Indirect : CMB measurement
with Λ CDM

Far universe, far past

Direct : original definition

$$cz = H_0 d$$

Near universe, near past

$$N_{\text{eff}} \quad 3.3-3.5 > 3.04$$

New particle ?

ニュートリノ振動

ニュートリノに質量を与える機構が必要

ミュオンの異常磁気能率

ミュオンの的なものに特異的に働く力が必要

IceCube Gap

ニュートリノに働く新しい力が必要

Hubble tension

ニュートリノ的な新しい粒子が必要

2. Phenomenology

- 2.1 IceCube Gap

(PRD 91 (2015) 3 037301, PRD 93 (2016) 7 076006)

- 2.2 Hubble Tension

(PTEP 2021 (2021) 10 103, arXiv 2306.?????)

4. A new gauged U(1): $L_\mu - L_\tau$

We introduce a new U(1) gauge symmetry associated with
the muon number minus tau number: $U(1)_{L_\mu - L_\tau}$.

$$\mathcal{L} = \mathcal{L}_{\text{SM}} - \frac{1}{4} Z'_{\rho\sigma} Z'^{\rho\sigma} - \cancel{\frac{\epsilon}{4} Z'_{\rho\sigma} B^{\rho\sigma}} + m_{Z'} Z'_\rho Z'^\rho$$

$$+ g_{Z'} (\bar{\nu}_\mu \gamma^\rho P_L \nu_\mu - \bar{\nu}_\tau \gamma^\rho P_L \nu_\tau + \bar{\mu} \gamma^\rho \mu - \bar{\tau} \gamma^\rho \tau) Z'_\rho$$

$L_\mu - L_\tau$

New gauge coupling

New gauge boson

1. No quantum gauge anomalies.
2. No LFV couplings.
3. Large atm. and small reactor mixing:
4. **A possible solution for muon anomaly** $\theta_{23} = 45^\circ$, $\theta_{13} = 0^\circ \therefore$

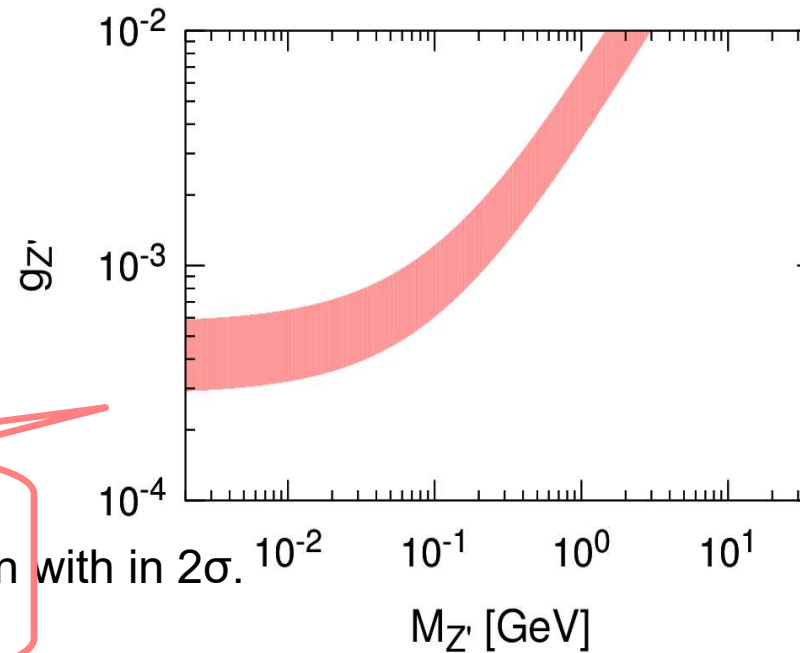
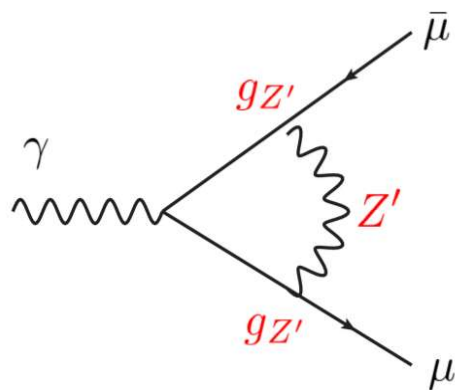
4.1 Muon g-2 discrepancy motivation to muonic force

Hagiwara, Liao, Martin, Nomura, Teubner, JPG38, 085003 (2011)

- The discrepancy between theory(SM) and experiment is

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (26.1 \pm 8.0) \cdot 10^{-10} (3.3\sigma)$$

- In $L_\mu-L_\tau$ models, we have an additional contribution.



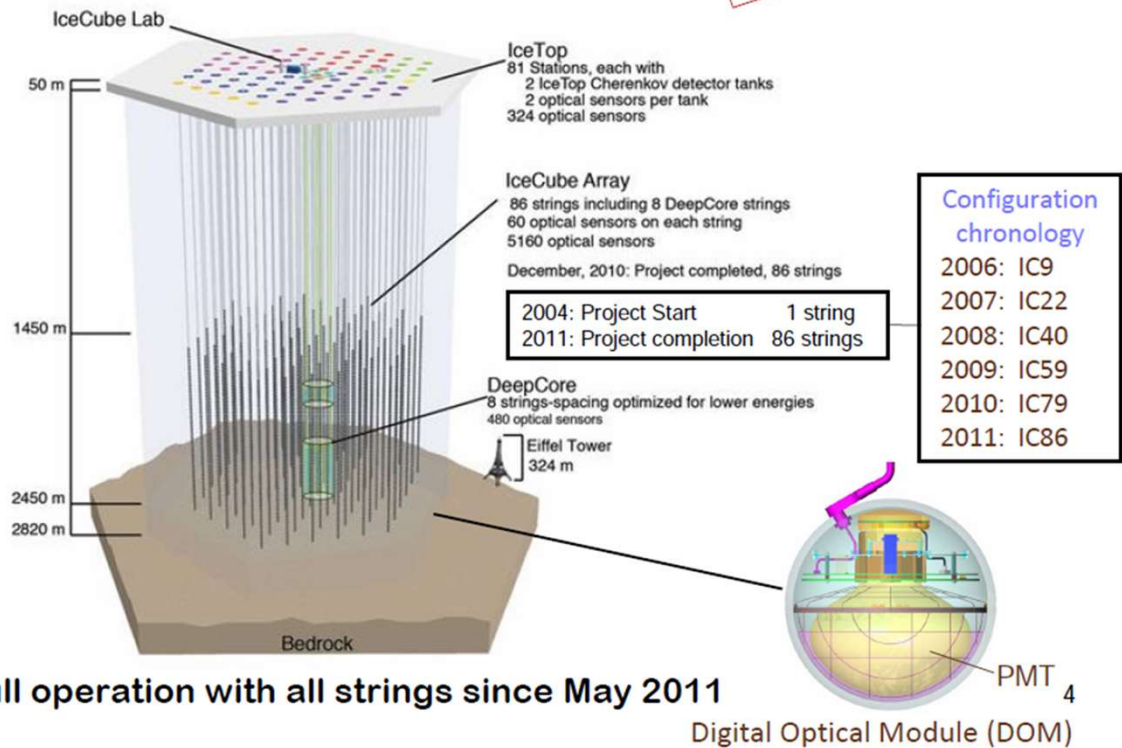
The discrepancy of muon $g-2$ can be explained in the red region with in 2σ .

4.2 IceCube Gap

The IceCube Neutrino Observatory

Completed: Dec 2010

Observe
High Energy Neutrinos
Including
Cosmic Neutrino



Full operation with all strings since May 2011

High energy cosmic neutrino

Target: Neutrinos produced in cosmic-ray interactions with gas (p) or radiation (γ), followed by pion decays.

@ Source

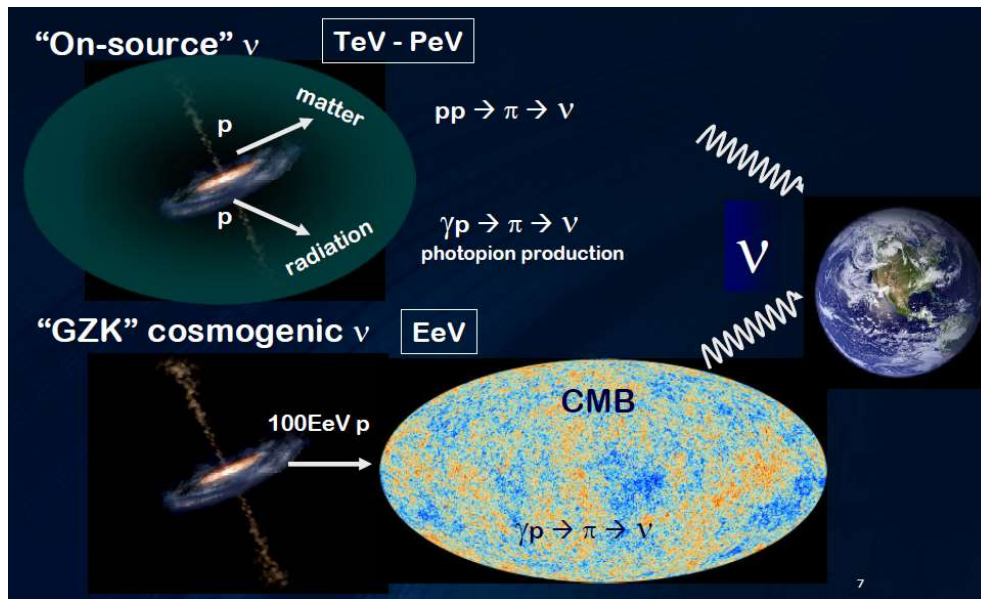
$$\nu_e : \nu_\mu : \nu_\tau$$

$$= 1 : 2 : 0$$

oscillation

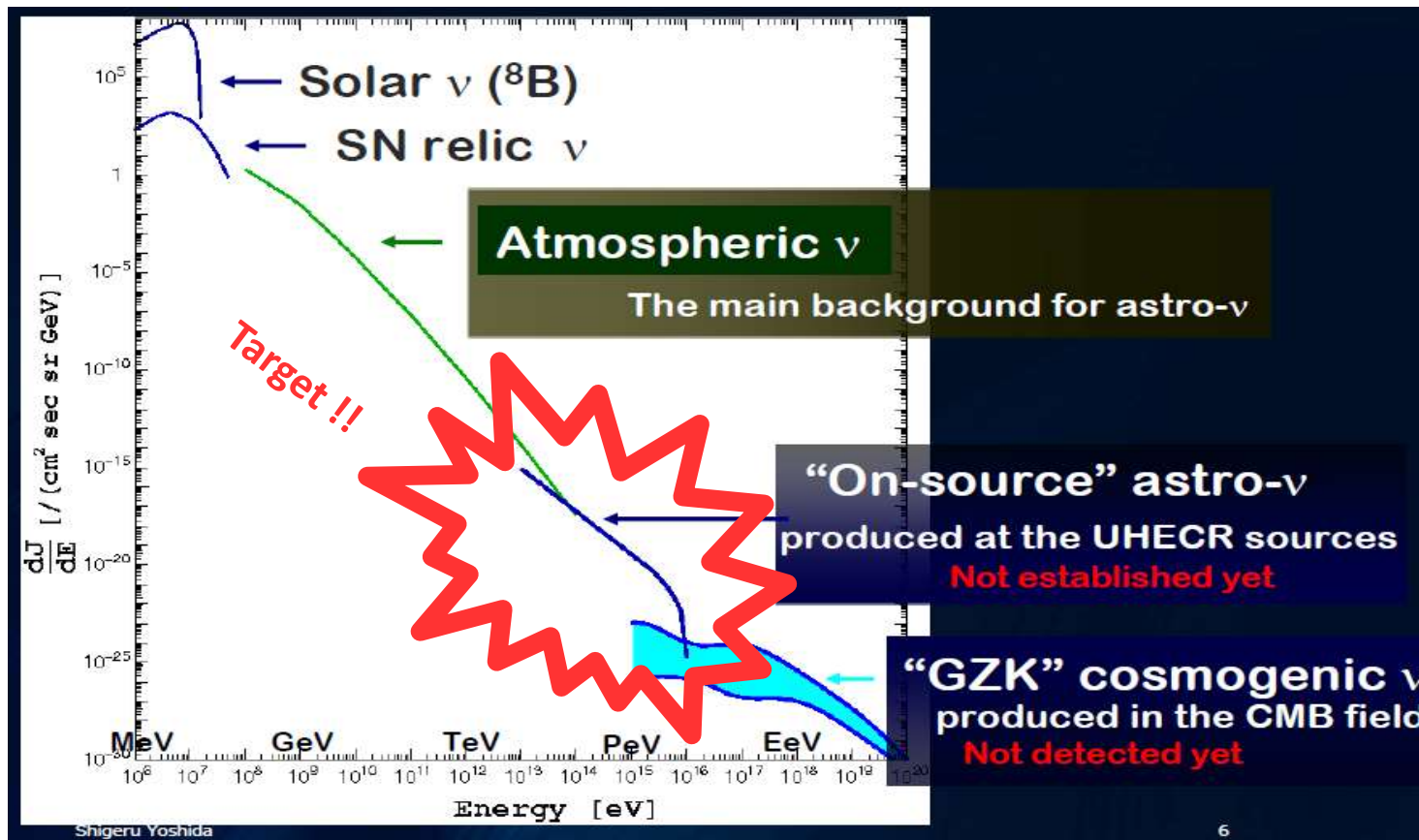
@ Earth

$$\simeq 1 : 1 : 1$$



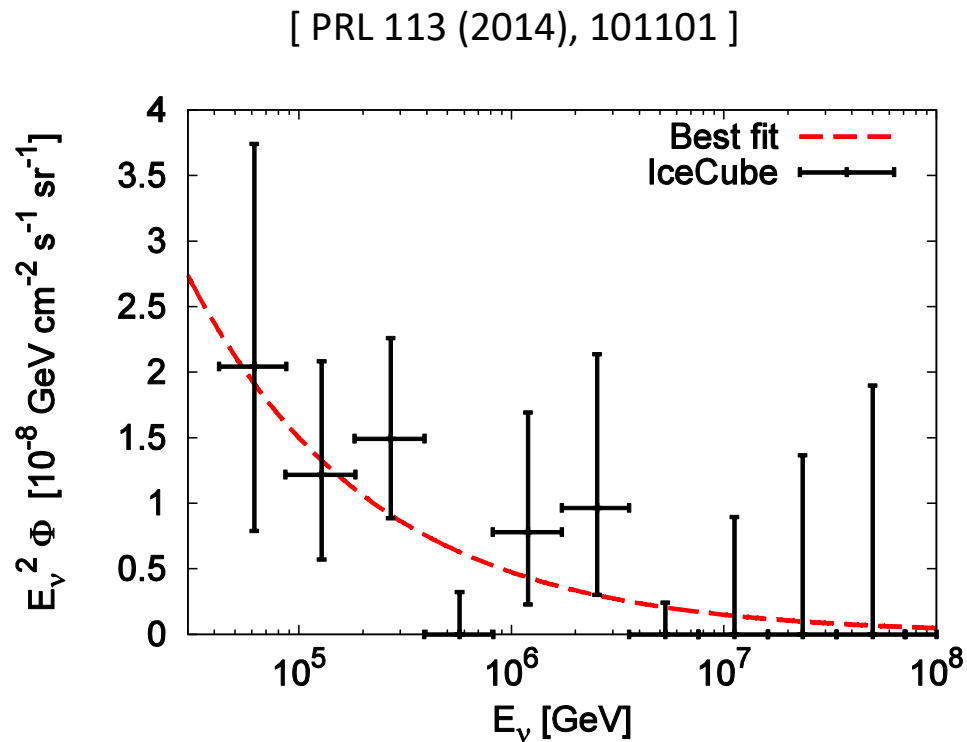
High energy cosmic neutrino

Target: Neutrinos having energies of O(TeV – PeV) .



Three-year data

Neutrino flux ($\nu + \bar{\nu}$) as a function of its energy.



1. It rejects a purely atmospheric explanation at **5.7** sigma.
2. The data are consistent with equal **(1:1:1)** flavor ratios and **isotropic** arrival directions.
3. The best-fit power law is

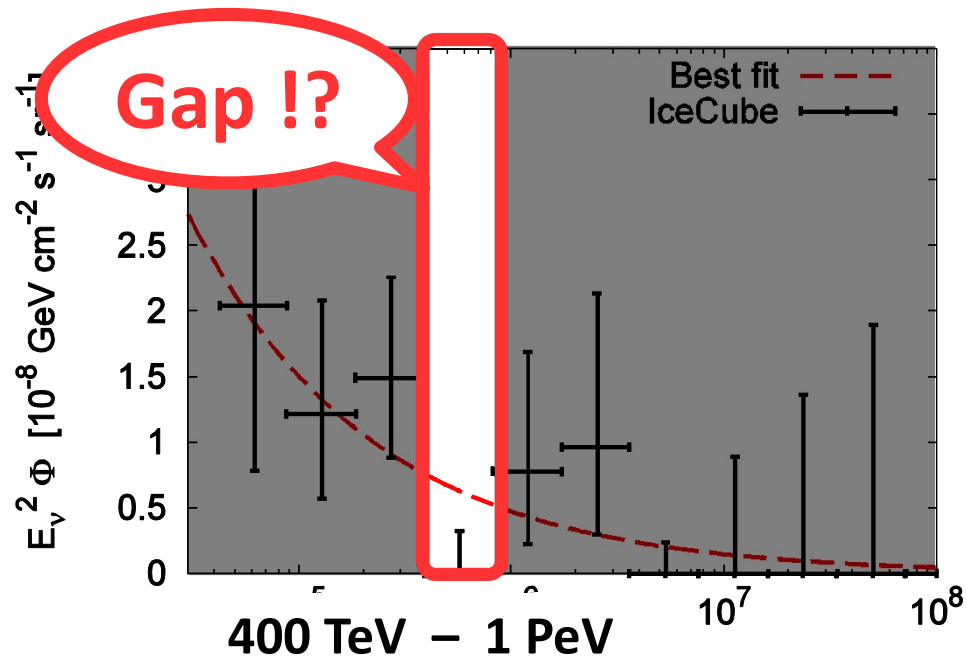
$$\Phi(E) = \phi \left[\frac{E_\nu}{100\text{TeV}} \right]^{-2.5}$$

※ Combined analysis:
[Astrophys. J. 809 (2015) 1, 98]

Gap or fluctuation?

Neutrino flux ($\nu + \bar{\nu}$) a function of its energy.

[PRL 113 (2014), 101101]



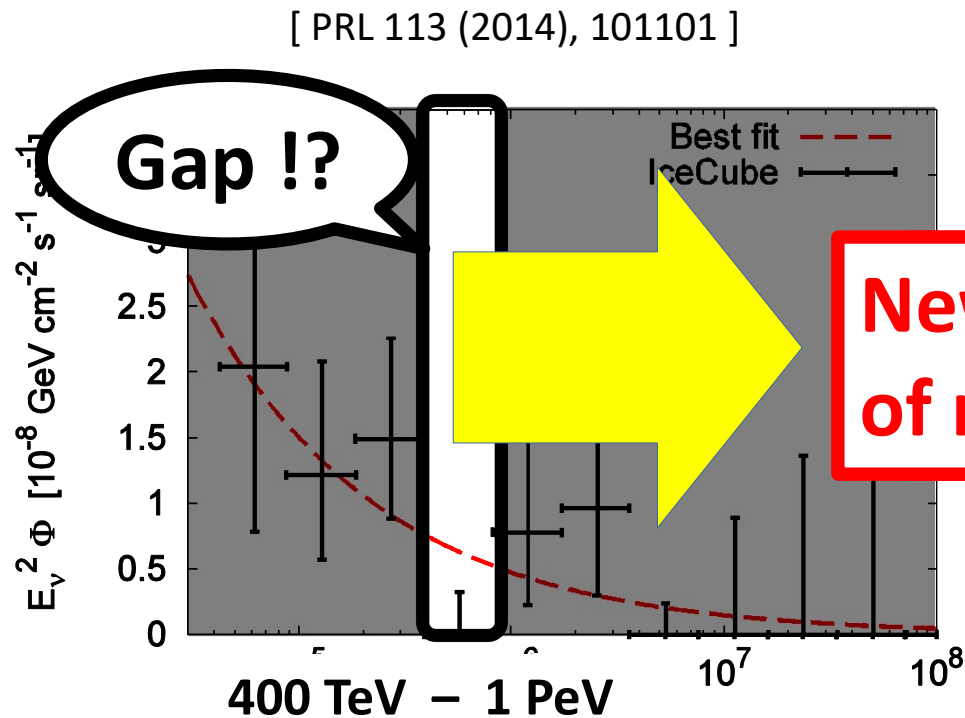
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New interactions of neutrinos ??

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[Astrophys. J. 809 (2015) 1, 98]

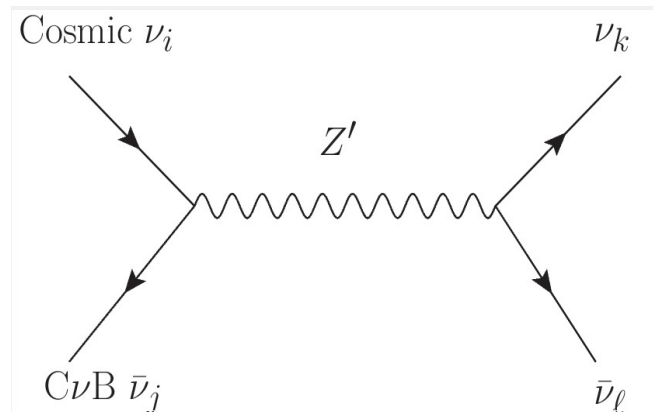
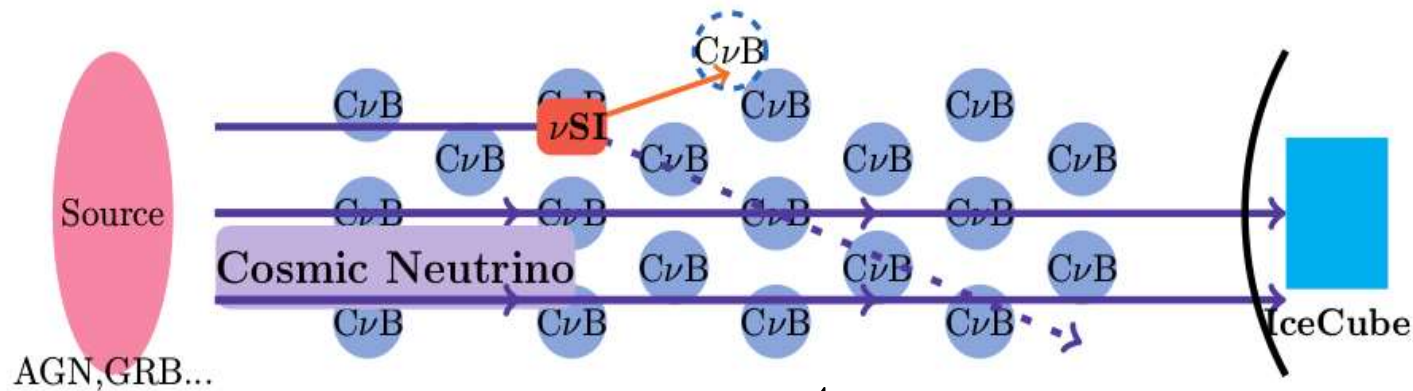
Ioka, Murase, PTEP2014, 061E01

Ng, Beacom, PRD90, 065035 (2014)

Ibe, Kaneta, PRD90, 053011 (2014)

Secret neutrino interaction

The Gap may indicate *Secret Neutrino Interaction (νSI)*.



\sqrt{s} : center of mass energy

$$\sigma = \frac{g_Z^4}{3\pi} \frac{s}{(s - M_{Z'}^2)^2 + M_{Z'}^2 \Gamma_{Z'}^2}$$

Cross section is enhanced as $\sqrt{s} \simeq M_{Z'}$

$$\sqrt{s} = \sqrt{(p_{\text{Cosmic}} + p_{\text{C}\nu\text{B}})^2}$$

$$\simeq \sqrt{2E_{\text{Cosmic}} m_\nu} \simeq \text{MeV}$$

500 TeV 10⁻³ eV

Secret neutrino interaction

Rough estimation of the mass of X and its coupling.

(1) M_X

**New physics at the MeV scale
is a possible candidate
for the IceCube gap!!**

ν
 $\bar{\nu}$

(2) To attenuate sufficient amount of cosmic neutrino:
 $m_{C\nu B} \simeq (0.01 - 0.1) \text{ eV} \quad E_\nu \simeq 1 \text{ PeV}$

$$\sigma > 10^{-30} \text{ cm}^2 \quad \rightarrow \quad \underline{g > 10^{-4}.}$$

A new gauged U(1): mu - tau

We introduce a new U(1) gauge symmetry associated with
the muon number minus tau number: $U(1)_{L_\mu - L_\tau}$.

$$\mathcal{L} = \mathcal{L}_{\text{SM}} - \frac{1}{4} Z'_{\rho\sigma} Z'^{\rho\sigma} - \cancel{\frac{\epsilon}{4} Z'_{\rho\sigma} B^{\rho\sigma}} + m_{Z'} Z'_\rho Z'^\rho$$

$$+ g_{Z'} (\bar{\nu}_\mu \gamma^\rho P_L \nu_\mu - \bar{\nu}_\tau \gamma^\rho P_L \nu_\tau + \bar{\mu} \gamma^\rho \mu - \bar{\tau} \gamma^\rho \tau) Z'_\rho$$

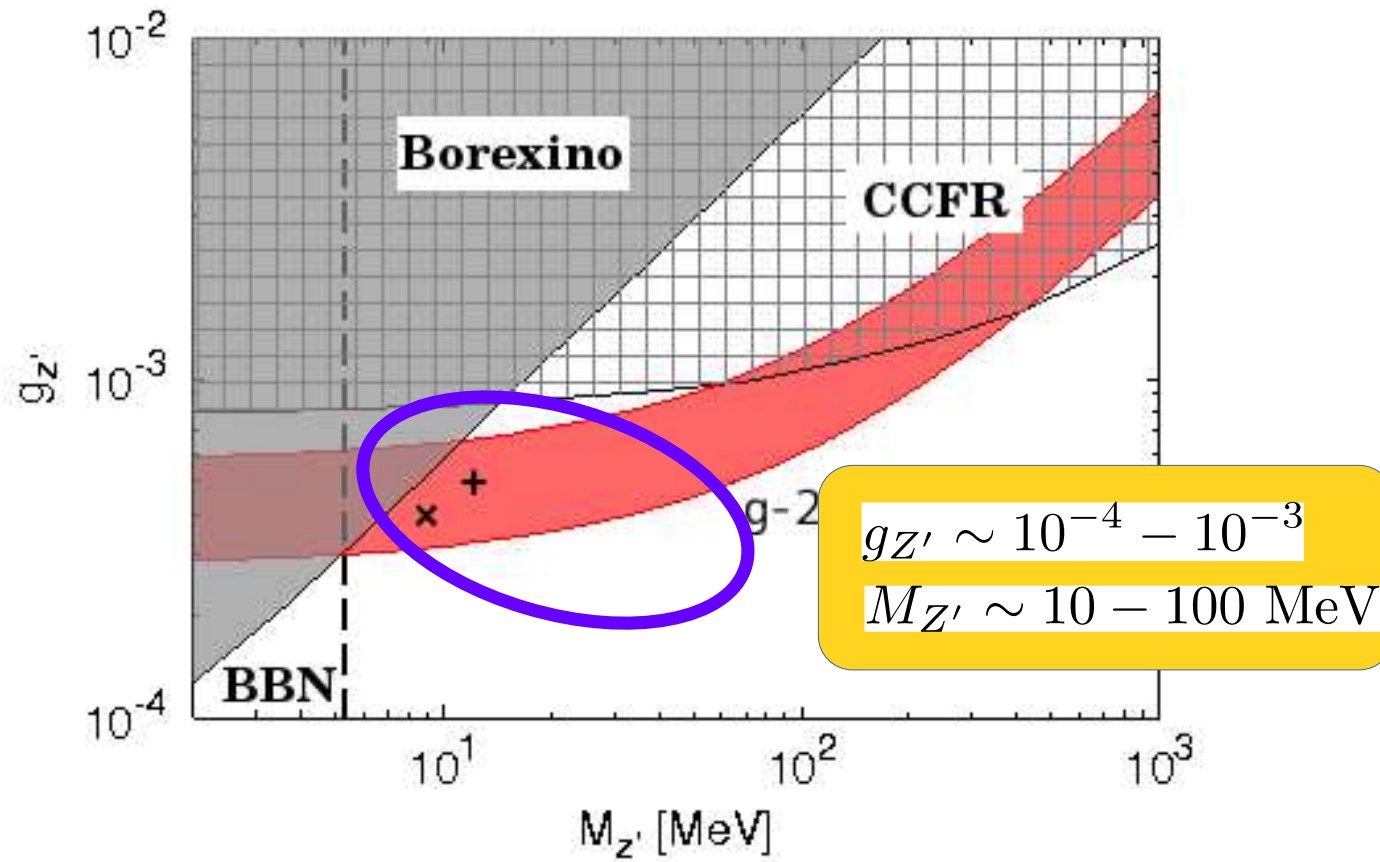
$$L_\mu - L_\tau$$

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Parameter region



Secret neutrino interaction

Rou Challenge ~~the mass of X and its coupling~~

**g-2 and IceCube gap
simultaneously ??**

(1) Resonant condition requires:

$$M_X \simeq \sqrt{2E_\nu^{\text{res}} m_{C\nu B}} \sim 1 - 10 \text{ MeV}.$$

$$m_{C\nu B} \simeq (0.01 - 0.1) \text{ eV} \quad E_\nu \simeq 1 \text{ PeV}$$

(2) To attenuate sufficient amount of cosmic neutrino:

$$\sigma > 10^{-30} \text{ cm}^2 \quad \rightarrow \quad \underline{g > 10^{-4}}.$$

Propagation of neutrinos

A propagation equation for cosmic neutrino:

$$\frac{\partial \tilde{n}_i}{\partial t} = \frac{\partial}{\partial E_i} b \tilde{n}_i + \mathcal{L}_i - c n_{\text{C}\nu\text{B}} \tilde{n}_i \sum_j \sigma(\nu_i \bar{\nu}_j^{\text{C}\nu\text{B}} \rightarrow \nu \bar{\nu})$$
$$+ c n_{\text{C}\nu\text{B}} \sum_{j,k} \int_{E_i}^{\infty} dE_k \tilde{n}_k \frac{d\sigma(\nu_k \bar{\nu}_j^{\text{C}\nu\text{B}} \rightarrow \nu_i \bar{\nu})}{dE_k}$$

$$\tilde{n}_i(E_i, z) = \frac{dn_i}{dE_i}$$

c: speed of light

z: redshift parameter

$n_{\text{C}\nu\text{B}}$: number density of CnB

Gap: Source distribution

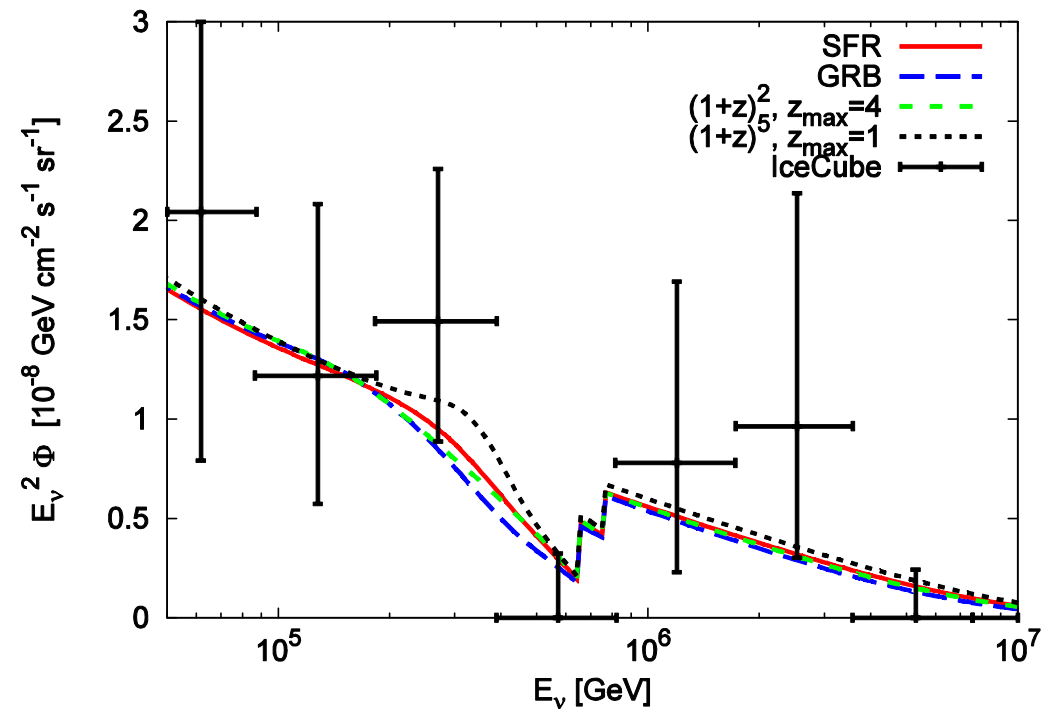
Diffuse neutrino flux for several types of source distribution.

Normal hierarchy

$m_1 = 0.08 \text{ eV}$
(quasi-degenerate)

$M_{Z'} = 11 \text{ MeV}$

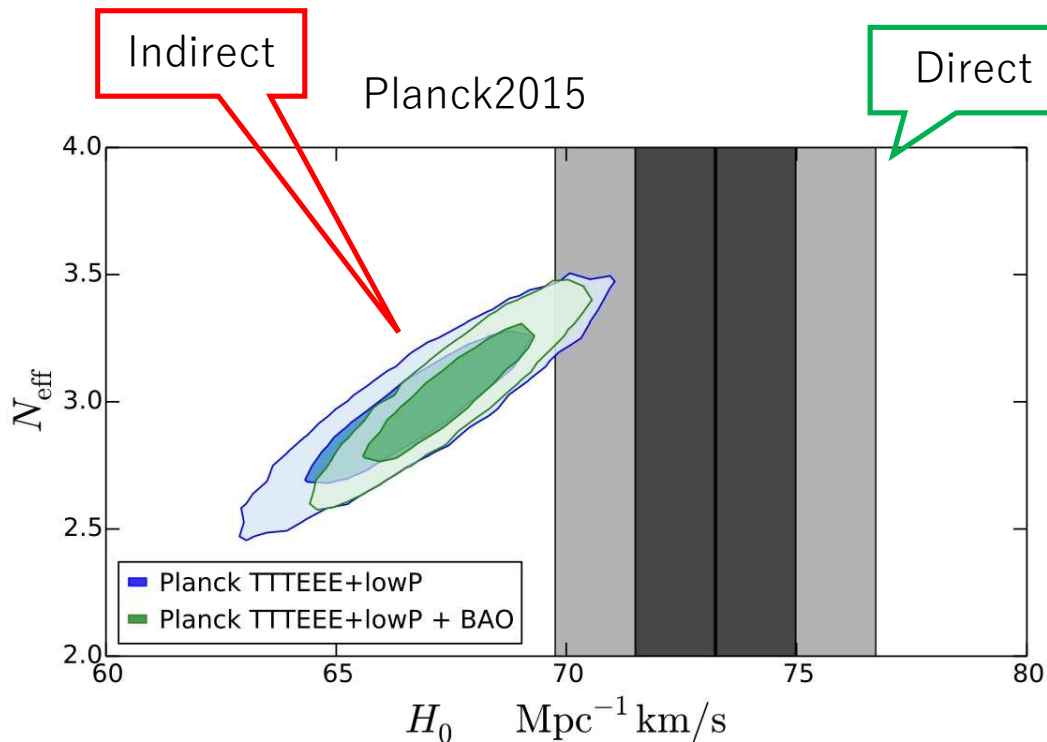
$g_{Z'} = 5 \times 10^{-4}$
($E_{\text{cut}} = 10^7 \text{ GeV}$)



Source distributions have **a small impact** on the flux.

4.3 Hubble Tension

Hubble Tension : Tension between two methods to measure H_0



José Luis Bernal, et al. Journal of Cosmology and
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Indirect : CMB measurement
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Far universe, far past

Direct : original definition

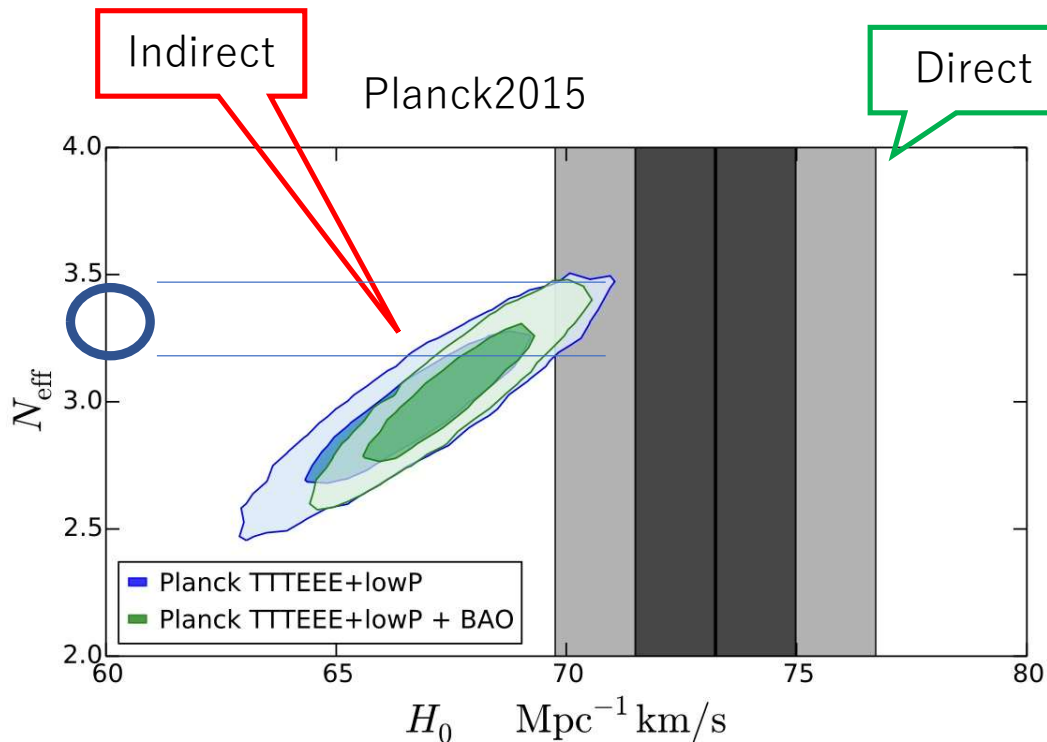
$$cz = H_0 d$$

Near universe, near past

In SM $N_{\text{eff}} = 3.04$

K.Akita and M.Yamaguchi Journal of Cosmology and
Astroparticle Physics, Vol. 2020, No. 08, p. 012 (2020)

Hubble Tension : Tension between two methods to measure H_0



José Luis Bernal, et al. Journal of Cosmology and
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Indirect : CMB measurement
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Far universe, far past

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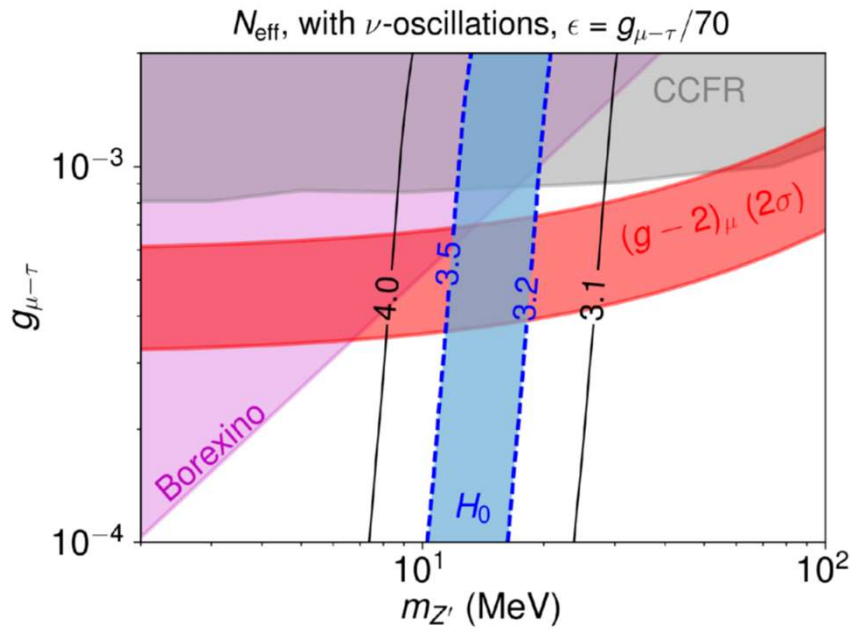
$$cz = H_0 d$$

Near universe, near past

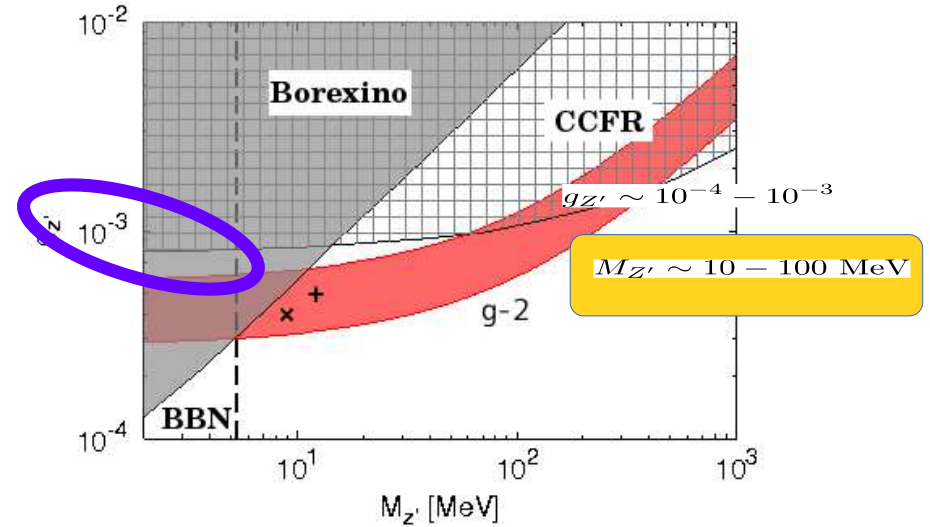
$$N_{\text{eff}} \quad 3.3-3.5 > 3.04$$

New particle ?

Solution by $L_\mu - L_\tau$ Gauge Boson

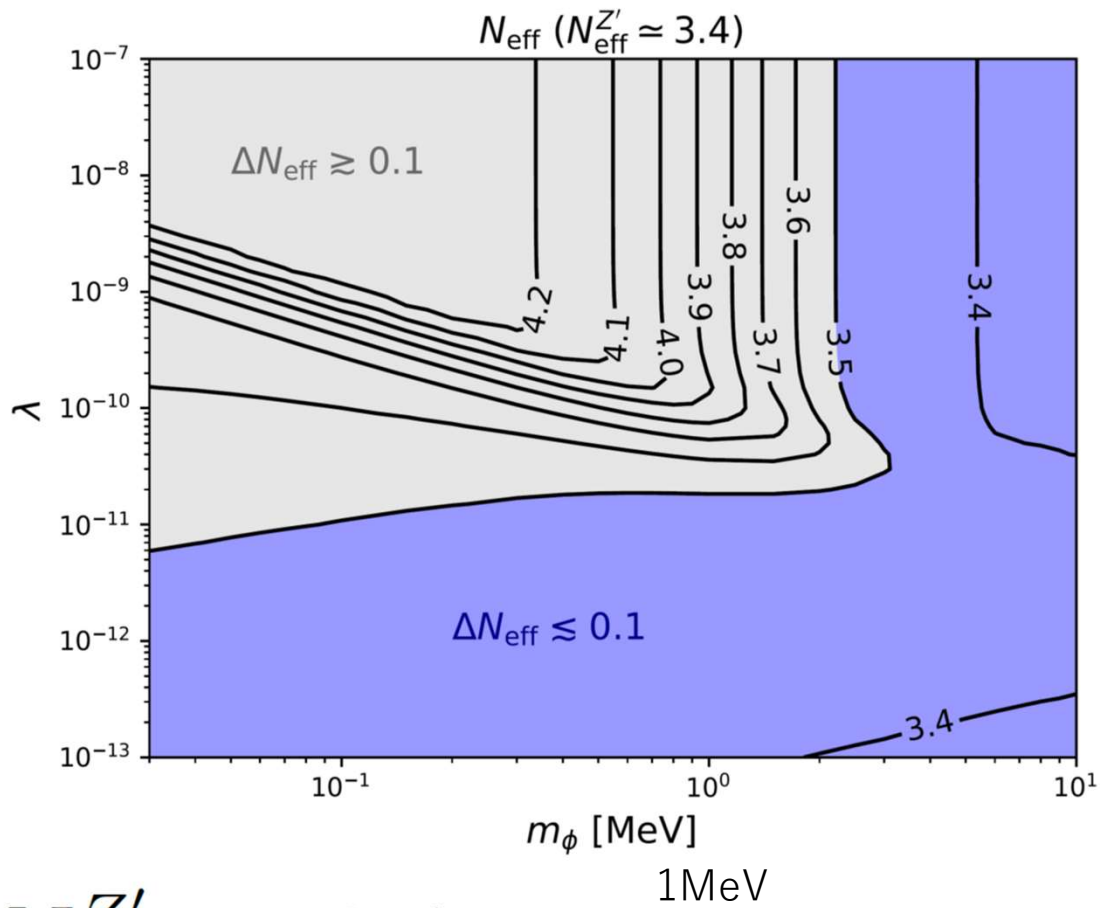


Cf.



M. Escudero, D. Hooper, G. Krnjaic, and M. Pierre,
 "Cosmology with A Very Light $L_\mu - L_\tau$ Gauge Boson,"
 JHEP 03 (2019) 071 [arXiv:1901.02010]

PRD 93 (2016) 7 076006



$$(m_{Z'}, g_{Z'}) \quad (13 \text{ MeV}, 5.0 \times 10^{-4})$$

Suitable for both g-2 and icecube
Majoron is included

Asai, Asano, JS, Yang, [2309.01162](https://arxiv.org/abs/2309.01162)

$$N_{\text{eff}}^{Z'} \approx 3.4$$

4.4. ニュートリノ質量とレプトン混合

- (PRD 100 (2019) 9 095012)

Renormalizable $L_\mu - L_\tau$ model see next subsection

$$G_{\text{SM}} \times U(1)_{L_\mu - L_\tau} \times U(1)_L$$

Lepton number $U(1)_L$
(Softly broken)

$U(1)_{L_\mu - L_\tau}$



Symmetry breaking
by scalar(s)



Light scalar ϕ (Majoron)

in addition to Z' mass

Renormalizable $\mathbf{L}_\mu - \mathbf{L}_\tau$ model

- \mathbf{Z}' interaction

$$\mathcal{L}_{Z'} = -\frac{1}{4} Z'^{\rho\sigma} Z'_{\rho\sigma} + \frac{1}{2} m_{Z'}^2 Z'^\rho Z'_\rho + g_{\mu-\tau} Z'_\rho J_{\mu-\tau}^\rho$$

$$Z'_{\rho\sigma} \equiv \partial_\rho Z'_\sigma - \partial_\sigma Z'_\rho$$

$$J_{\mu-\tau}^\rho \equiv \bar{\mu} \gamma^\rho \mu + \bar{\nu}_\mu \gamma^\rho P_L \nu_\mu - \bar{\tau} \gamma^\rho \tau - \bar{\nu}_\tau \gamma^\rho P_L \nu_\tau$$

- ϕ

$$\mathcal{L}_\phi = h_{\alpha\beta} \bar{\nu}_{L,\alpha} \nu_{L,\beta}^c \phi + h.c.$$

- $Z' \leftrightarrow \nu_{\alpha'} \bar{\nu}_{\alpha'}$ ($\alpha' = \mu, \tau$)
- $\phi \leftrightarrow \nu_\alpha \nu_\beta$
- $Z' \nu_\alpha \leftrightarrow \phi \bar{\nu}_\beta$ ($\nu \leftrightarrow \bar{\nu}$ もあり)
- $Z' \phi \leftrightarrow \nu_\alpha \nu_\beta$ ($\nu \leftrightarrow \bar{\nu}$ もあり)

Should be included

Large $\mu - \tau$ mixing from neutrino oscillation



Suggesting a symmetry between $\mu - \tau$ flavor !?



$L_\mu - L_\tau$ is one of such a symmetry.

Indeed many physicists make models based on it to explain neutrino mass and lepton mixing !!

Zero texture and large mixing

Merits of the symmetry

Mass matrix in $U(1)_{L_\mu - L_\tau}$

- $U(1)_{L_\mu - L_\tau}$ charges of a Majorana mass matrix.

$$Q_{L_\mu - L_\tau}(M_{\text{Majorana}}) = \begin{pmatrix} \textcircled{0} & \boxed{1} & \boxed{-1} \\ \boxed{1} & \textcircled{2} & \textcircled{0} \\ \boxed{-1} & \textcircled{0} & \textcircled{-2} \end{pmatrix}$$

- Thus, the mass matrix is given by

$$M_{\text{Majorana}} = \begin{pmatrix} \times & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix}$$



Large mixing

Zero texture and large mixing

Mass matrix in $U(1)_{L_\mu - L_\tau}$

- $U(1)_{L_\mu - L_\tau}$ charges of a Majorana mass matrix.

$$Q_{L_\mu - L_\tau}(M_{\text{Majorana}}) = \begin{pmatrix} \textcircled{0} & \boxed{1} & \boxed{-1} \\ \boxed{1} & \boxed{2} & \textcircled{0} \\ -1 & \textcircled{0} & \boxed{-2} \end{pmatrix}$$

- Thus, the mass matrix is given by

$$M_{\text{Majorana}} = \begin{pmatrix} \times & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix} \xrightarrow{U(1)_{L_\mu - L_\tau}} M_{\text{Majorana}} = \begin{pmatrix} \times & 0 & \times \\ 0 & 0 & \times \\ \times & \times & 0 \end{pmatrix}$$

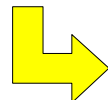
(ex.) $\bar{\Psi}_R^c \Psi_R \phi_1$

Zero textures

Zeros in neutrino mass matrix gives constraint = prediction

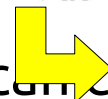
- One zero yields two conditions.
- Zero textures

$$(M_\nu)_{ab} = 0$$


$$(m_1 e^{2i\alpha_1}) U_{a1} U_{b1} + (m_2 e^{2i\alpha_2}) U_{a2} U_{b2} + (m_3) U_{a3} U_{b3} = 0$$

- Zero-minor textures

$$(M_\nu^{-1})_{ab} = 0$$

- One can check  $\frac{1}{m_1 e^{2i\alpha_1}} U_{a1} U_{b1} + \frac{1}{m_2 e^{2i\alpha_2}} U_{a2} U_{b2} + \frac{1}{m_3} U_{a3} U_{b3} = 0$

Testable!

$$M_\nu = V \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} V^T$$
$$V_{\text{MNS}} = U \cdot \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$


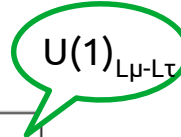
Our model

Majoron here !

Particle contents

	$\ell_{Le}, \ell_{L\mu}, \ell_{L\tau}$	$\ell_{Re}, \ell_{R\mu}, \ell_{R\tau}$	$N_{Re}, N_{R\mu(\tau)}$	$N_{Le}, N_{L\mu(\tau)}$	H	Φ	S_L	$S_{\mu\tau}$
$U(1)_{L\mu-L\tau}$	0, 1, -1	0, 1, -1	0, 1(-1)	0, 1(-1)	0	0	0	1
$U(1)_L$	1	1	1	1	0	-2	-2	0
$SU(2)_L$	2	1	1	1	2	2	1	1

2 generations

▣ Model – A : $(N_{Re}, N_{R\mu}) = (0, 1)$ $(N_{Le}, N_{L\mu}) = (0, 1)$

▣ Model – B : $(N_{Re}, N_{R\tau}) = (0, -1)$ $(N_{Le}, N_{L\tau}) = (0, -1)$

5. Summary

$L_\mu - L_\tau$ is very interesting in phenomenology, explaining

muon anomalous magnetic moment
IceCube Gap
Hubble tension etc.

Probably relevant with neutrino mass and lepton mixing

It is a gauged family symmetry

Suggesting Grand-Grand Unification , gauge and family unification

Candidate framework

Nonlinear supersymmetric realization ?, Coset Space Dimensional Reduction ?
CosetSpace structure seems interesting