Implementation of the tools of physics in scenarios involving economics and finance

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Talk based on *Physica A* 526 (2019) 121028; *EPL* 131 (2020) 6, 68003; Mathematics 2021, 9, 2777 and other papers to be mentioned during the talk.

Hamiltonian formulation: Generalities

- The Hamiltonian formulation is general. It can be used in any area of science.

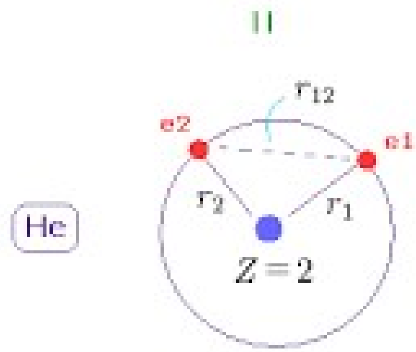
- The Hamiltonian and Lagrangian formulation are equivalent. Using one or the other is a matter of convenience.

$$\mathcal{H} = \sum_i \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi_i} \partial_\mu \phi_i - \mathcal{L}$$

Hamiltonian formulation

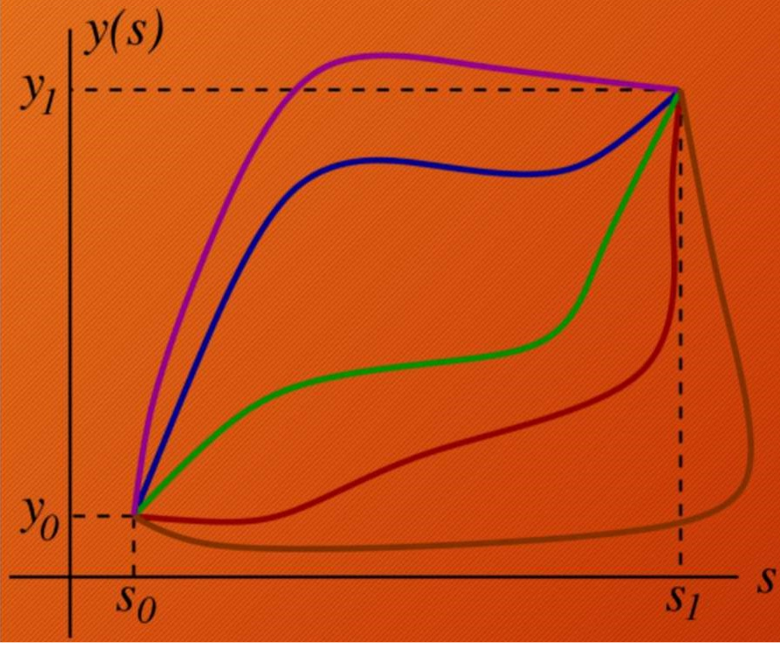
- It is the favorite for explaining results related to atomic physics. Here Relativistic effects can be normally neglected.

$$\hat{H}\phi_0 = \left(-\frac{\hbar^2}{2m_e}\nabla_1^2 - \frac{\hbar^2}{2m_e}\nabla_2^2 - \frac{Ze^2}{4\pi\epsilon_0 r_1} - \frac{Ze^2}{4\pi\epsilon_0 r_2} + \frac{e^2}{4\pi\epsilon_0 r_{12}}\right)\phi_0$$



Lagrangian formulation

- It is the favorite for explaining problems related to High-Energy Physics. The invariance of the Lagrangian is the key ingredient here.

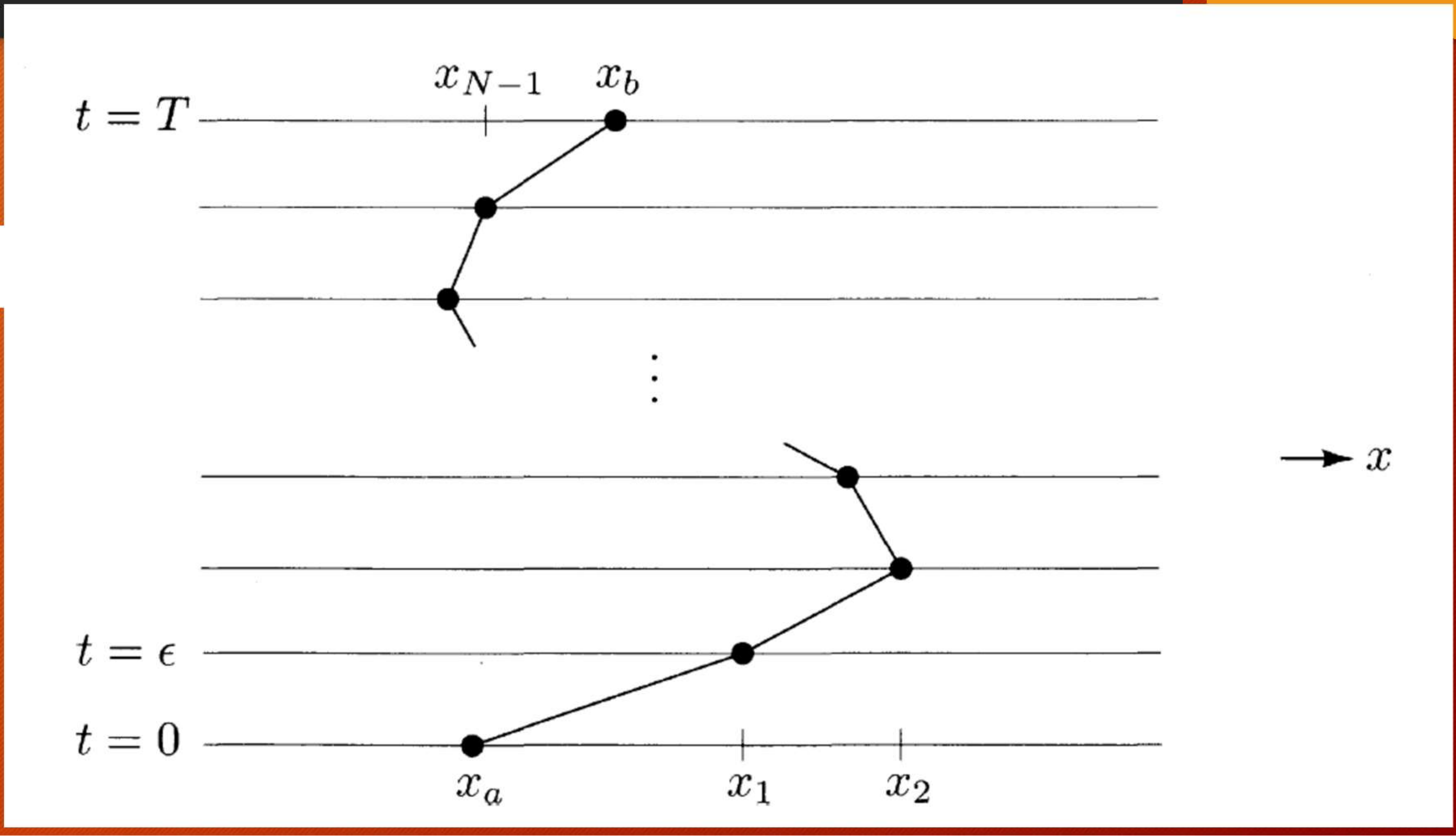


How did these formulations were interpreted?

- Feynman

$$U(x_a, x_b; T) = \langle x_b | \, e^{-iHT/\hbar} \, | x_a \rangle \, .$$

Amplitude of propagation!!



Double slit experiment: The best example!!!

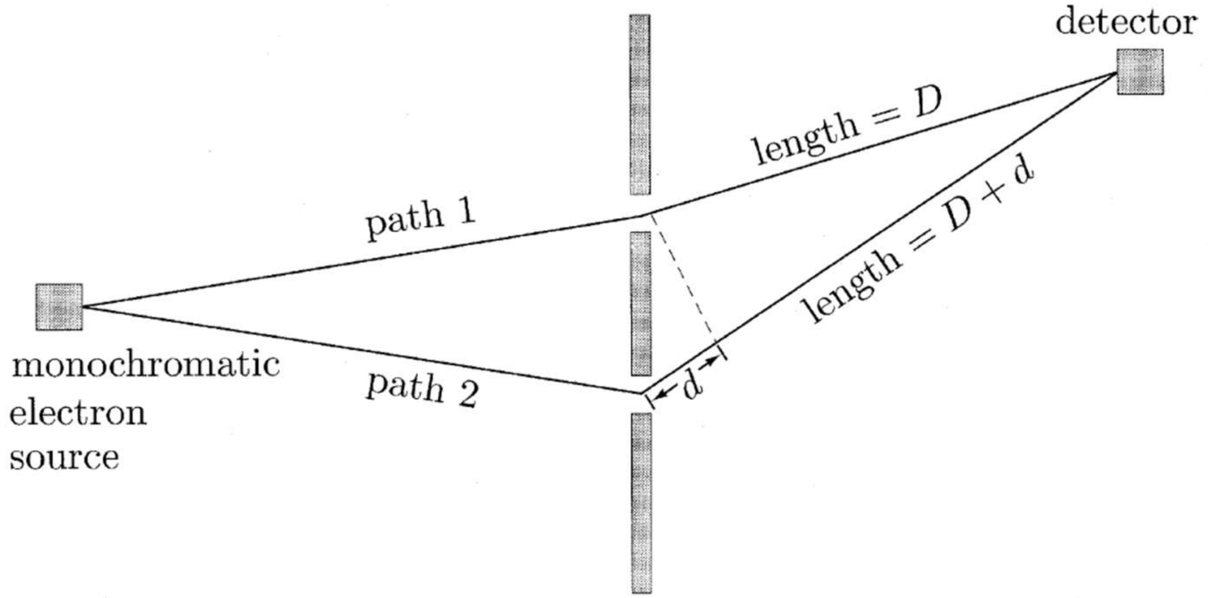
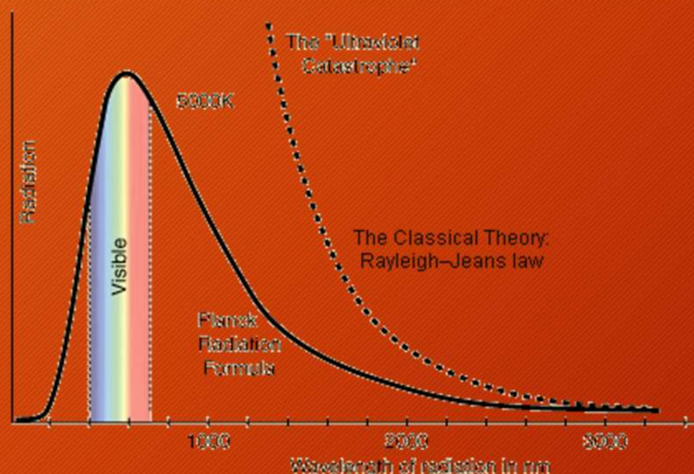


Figure 9.1. The double-slit experiment. Path 2 is longer than path 1 by an amount d, and therefore has a phase that is larger by $2\pi d/\lambda$, where $\lambda = 2\pi\hbar/p$ is the particle's de Broglie wavelength. Constructive interference occurs when $d = 0,\ \lambda, \ldots,$ while destructive interference occurs when $d = \lambda/2,\ 3\lambda/2, \ldots.$

Taken from Peskin
And Schroeder

THE NECESSITY OF QUANTUM MECHANICS

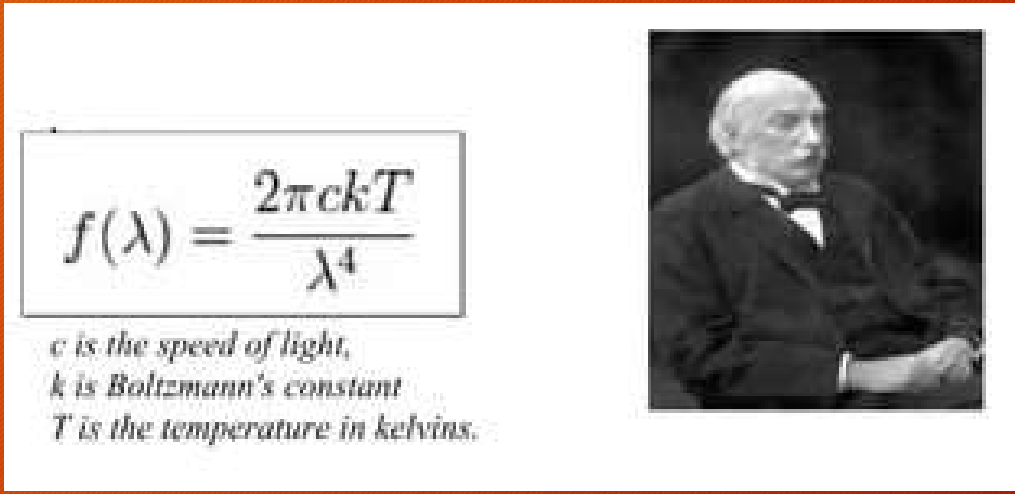
- THE UV CATASTROPHE:



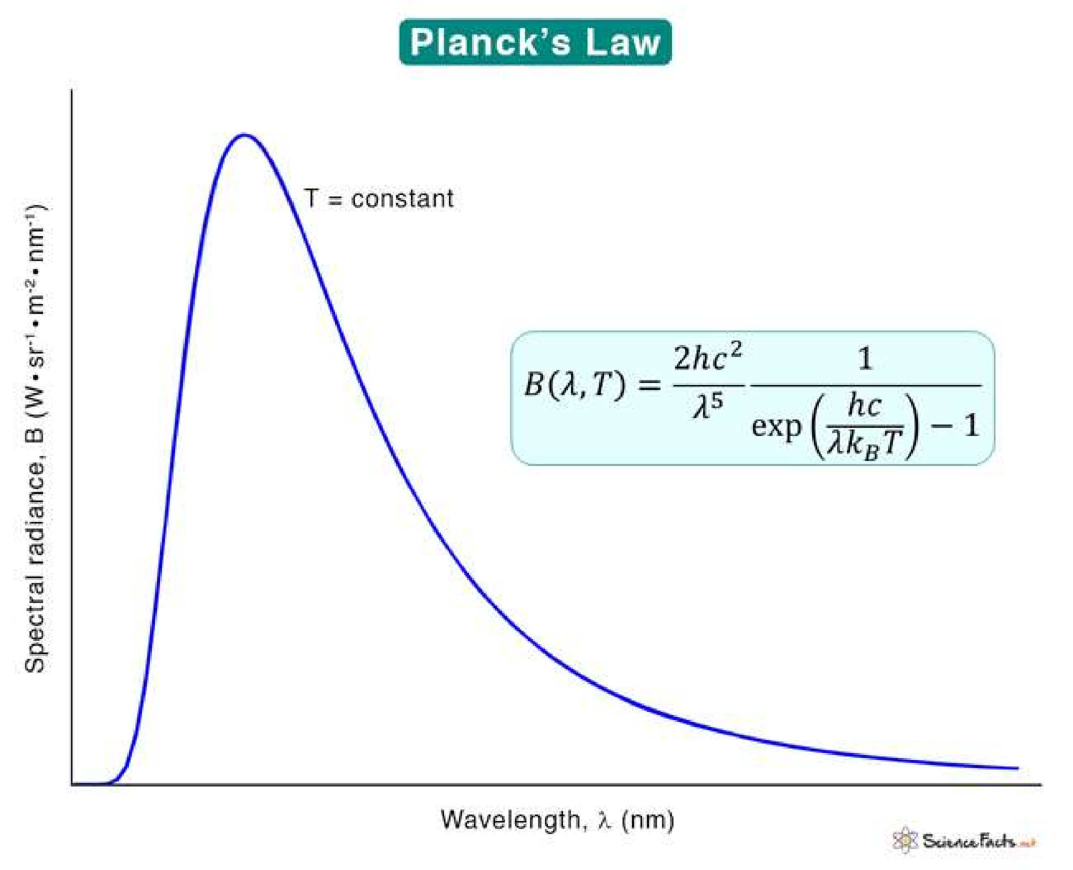
The Rayleight-Jeans catastrophe. Max Planck arranged in in 1900 by doing an incredible assumption.

Assumptions of Planck

- 1). Energy appeared in small packages…Subsequently known as quanta. Just take a look at the Rayleigh-Jeans law, which did not consider this aspect

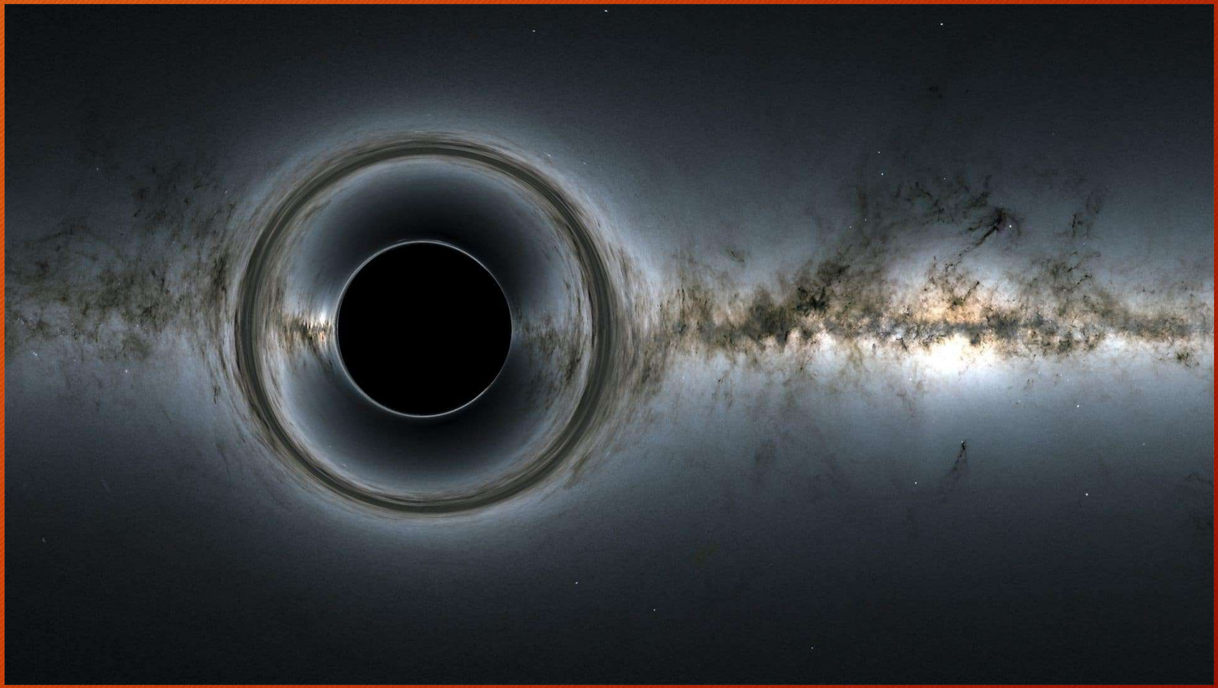


Compare with Planck's expressions



Guess what?

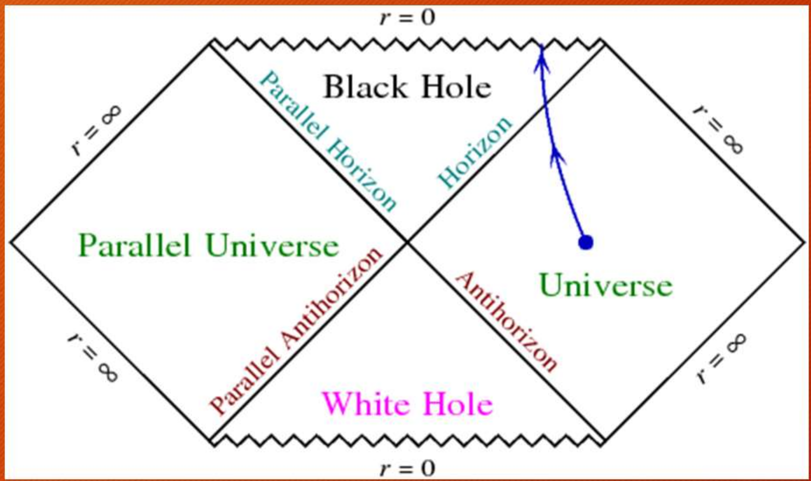
- Hawking found similar radiation for the Black-Hole spectrum of radiation.



$$<\bar{0}|\hat{n}^a_{\mathbf{p}}|\bar{0}>=\frac{\Gamma_{\mathbf{p},\mathbf{p}'}}{e^{\frac{2\pi\omega}{\kappa}}\pm 1}.$$

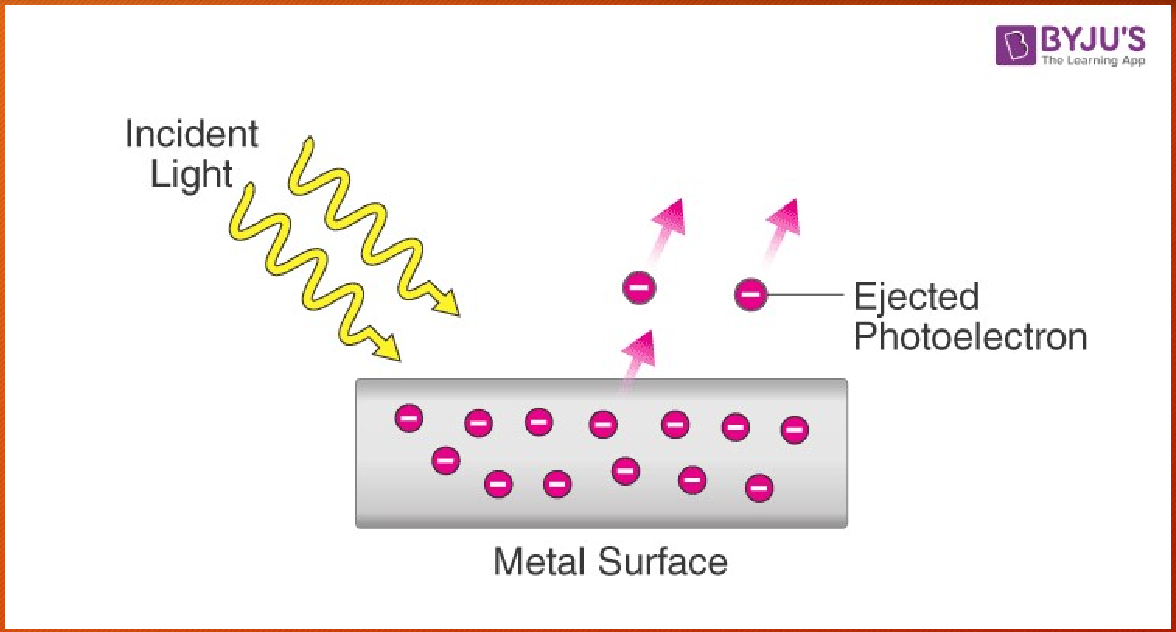
For fermions and bosons.

How can this appears in Black Hole Physics (Penrose diagrams!!!)



- The gravitational collapse problem. Mixing Quantum Mechanics and General Relativity.

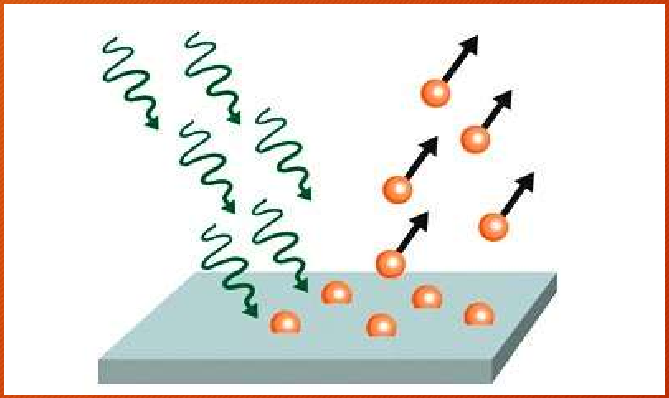
Einstein proposed the concept of Quanta (Photoelectric effect).

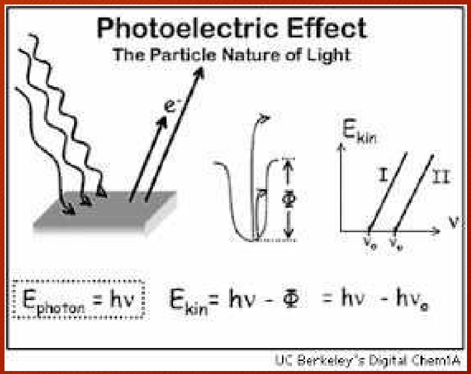


The photoelectric effect phenomena was not well-understood before introducing the concept of Quanta.

Key points of the photoelectric effect

- 1). There is a minimal energy, after which an electron can really move away and transport current.
- 2). The intensity of the light does not affect the kinetic energy of the electrons.
- 3). The kinetic energy is only affected by the frequency of the light.





EMERGENCE OF QUANTUM MECHANICS

- Heisenberg vs Schrodinger formulation:
- The Schrodinger formulation considers the solution of a wave function, which describes the motion of a particle or the evolution of a system.
-

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi$$

Heisenberg formulation

Heisenberg equation of motion

Schrodinger picture

$$\langle O(t)\rangle = \overbrace{\langle\Psi|U^{\dagger}(t-t_0)}\,O(t_0)\,\overbrace{U(t-t_0)|\Psi\rangle}$$

$$\langle\Psi|\underbrace{U^{\dagger}(t-t_0)\,O(t_0)\,U(t-t_0)}|\Psi\rangle$$

Heisenberg picture

Heisenberg equation of motion

$$\frac{d}{dt}X(t) = \frac{1}{i\hbar}[X(t), H] \qquad \frac{d}{dt}P(t) = \frac{1}{i\hbar}[P(t), H]$$

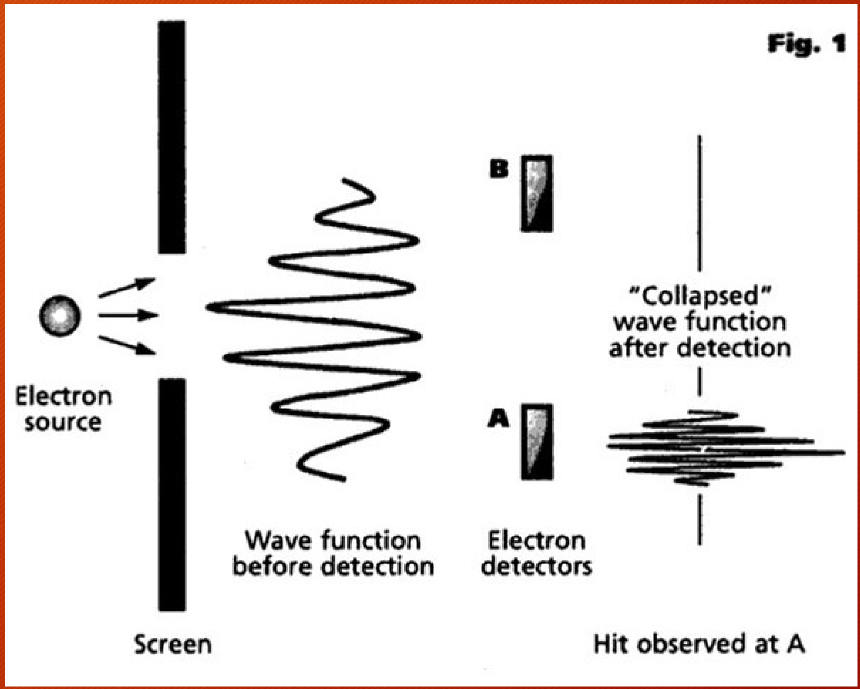
$$\frac{d}{dt}O = \frac{1}{i\hbar}[O, H] + \frac{\partial}{\partial t}O$$

KEY POSTULATES OF QUANTUM MECHANICS

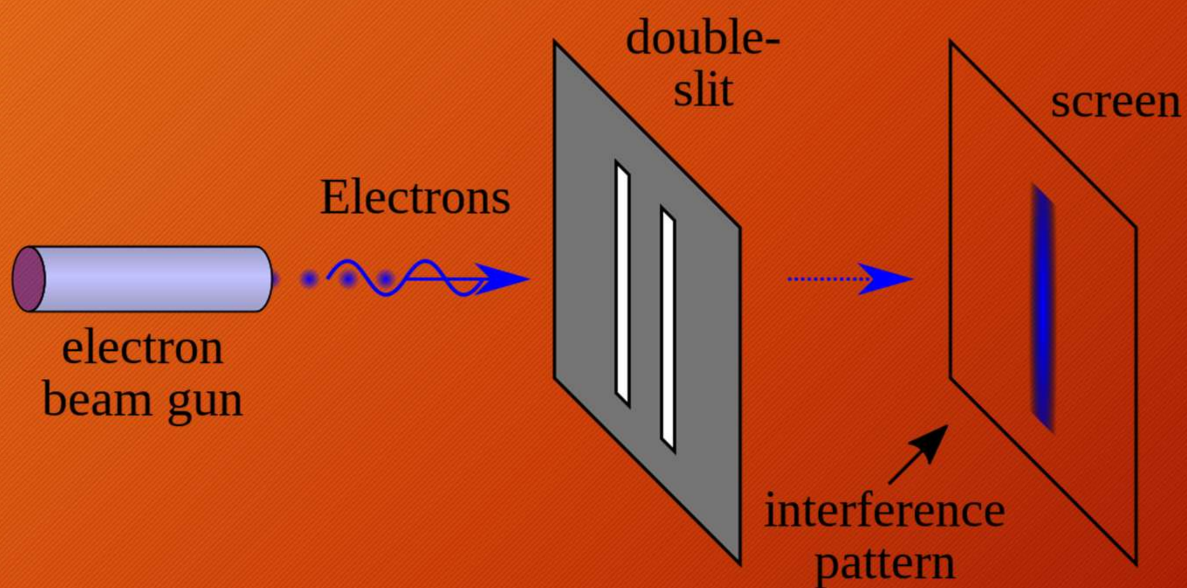
- 1). Uncertainty principle: It is impossible to know position and momentum simultaneously.

$$\Delta x \, \Delta p \geq \frac{\hbar}{2}$$

- 2). Complementarity:

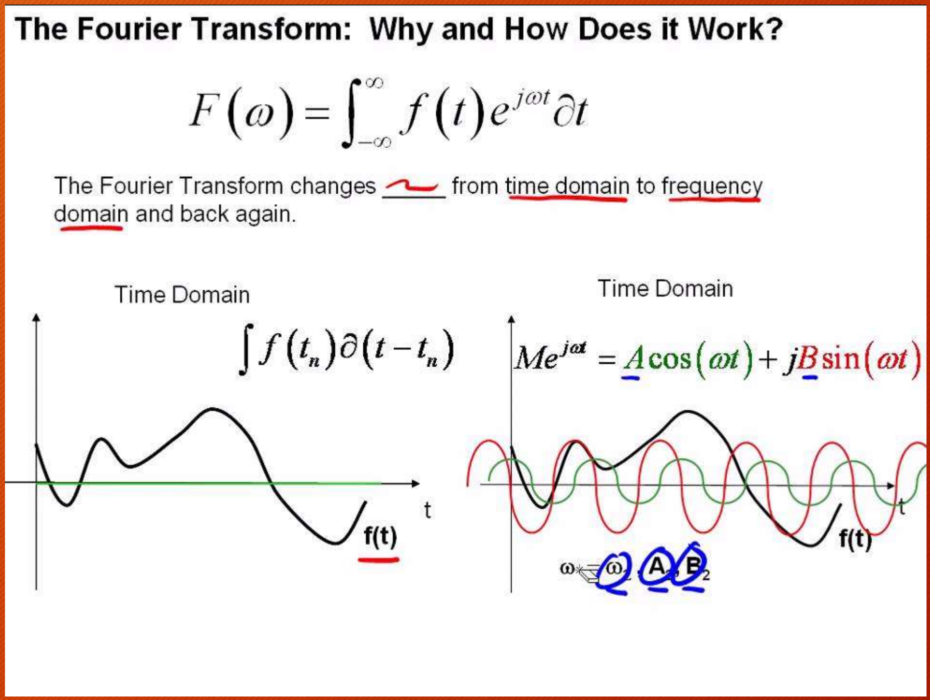


The double slit experiment!!!



MATHEMATICAL UNDERSTANDING OF THE UNCERTAINTY PRINCIPLE

- Fourier transformation:



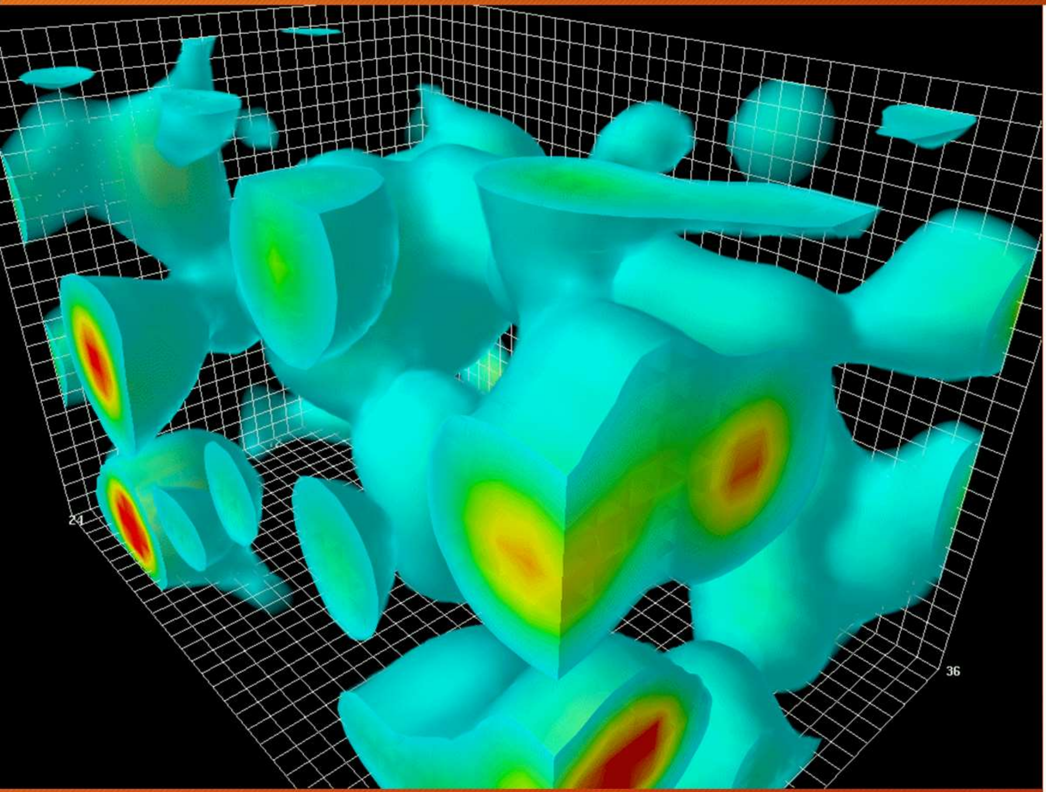
FOURIER TRANSFORMATION IN QM

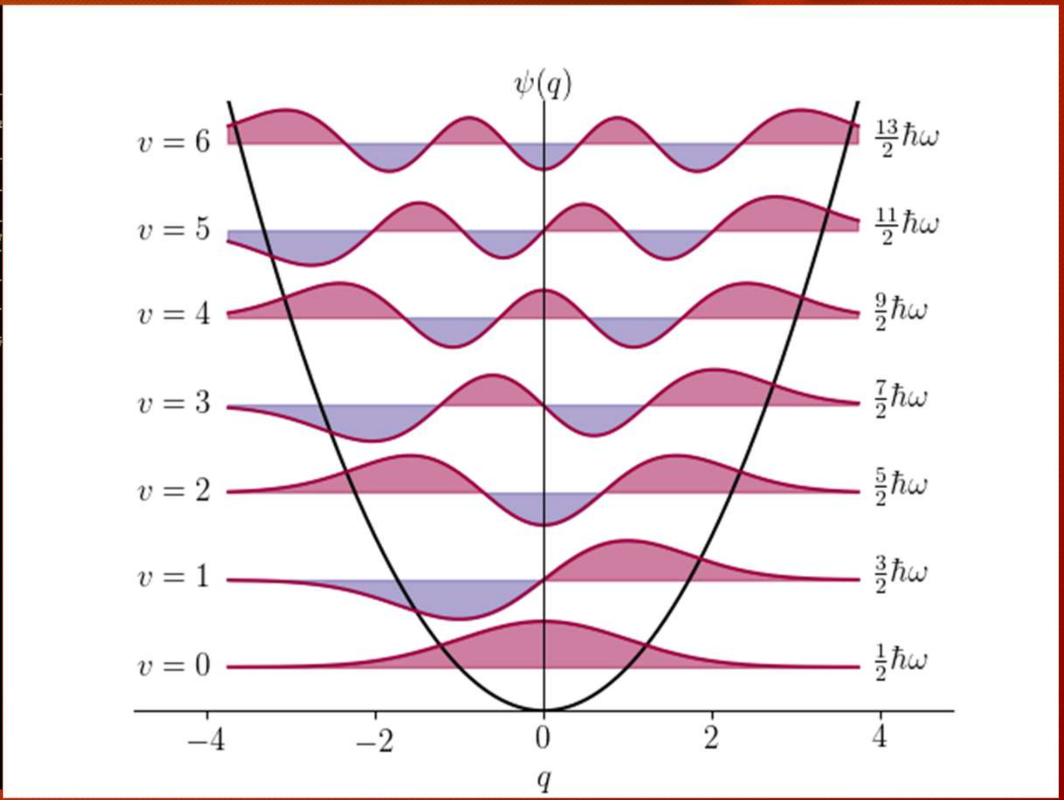
- In QM, we have to decide in which space do we work. This limits our capability of knowledge for certain variables. We have to mnake a compromise based on the Fourier transform (Uncertainty principle).

$$\Psi(x) \;=\; \frac{1}{\sqrt{a\sqrt{\pi}}}e^{-(x/2a)^2}$$

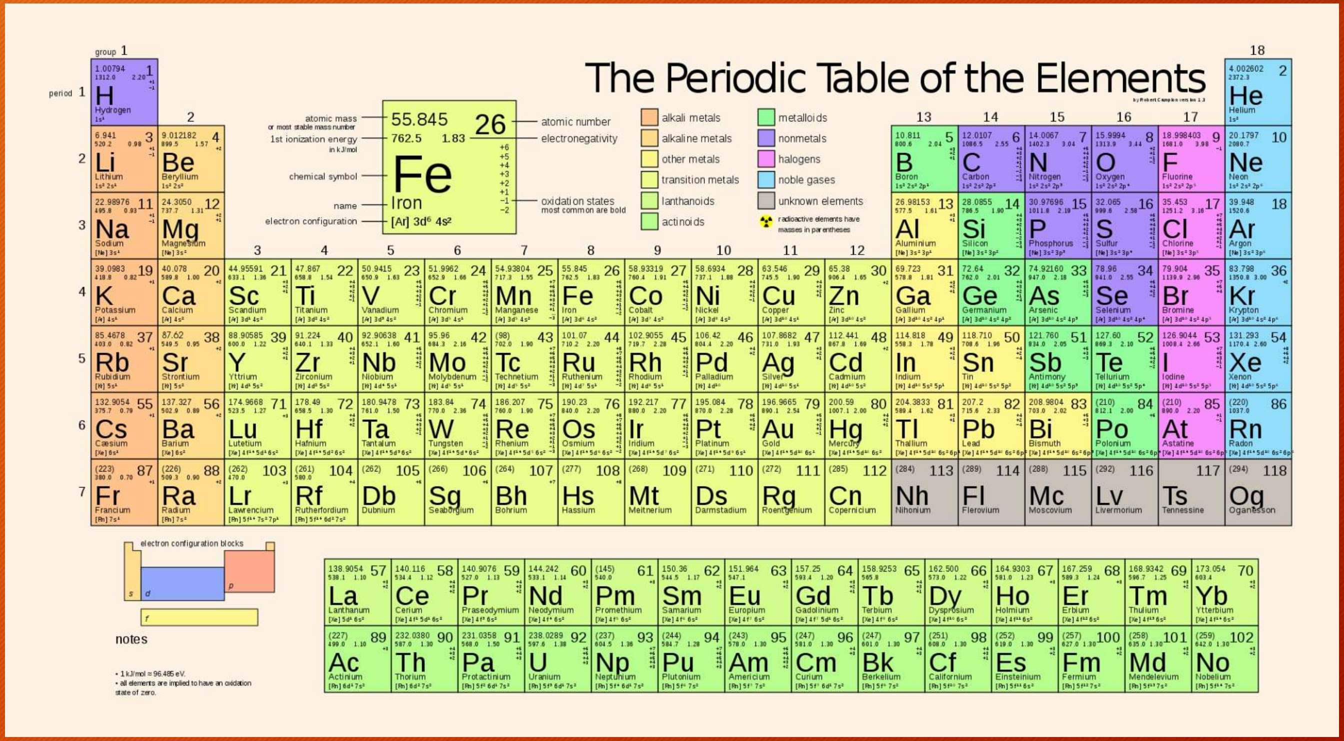
$$\Phi(p) \;=\; \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\Psi(x)e^{-ipx}\,\mathrm{d}x$$

UNCERTAINTY PRINCIPLE AND VACUUM ENERGY

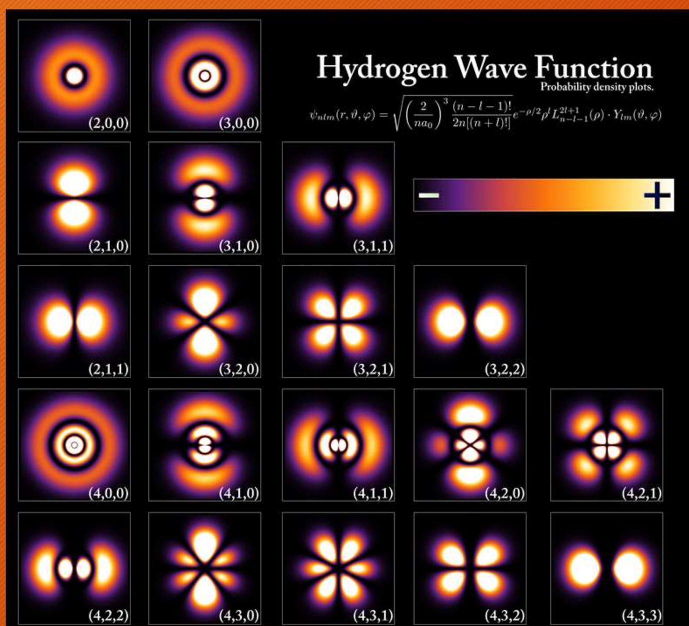




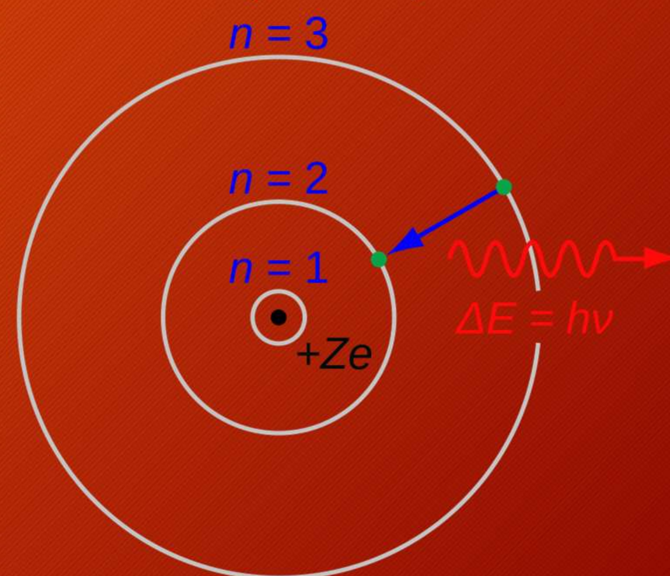
QUANTUM MECHANICS IN CHEMISTRY



QUANTUM MECHANICS IN CHEMISTRY



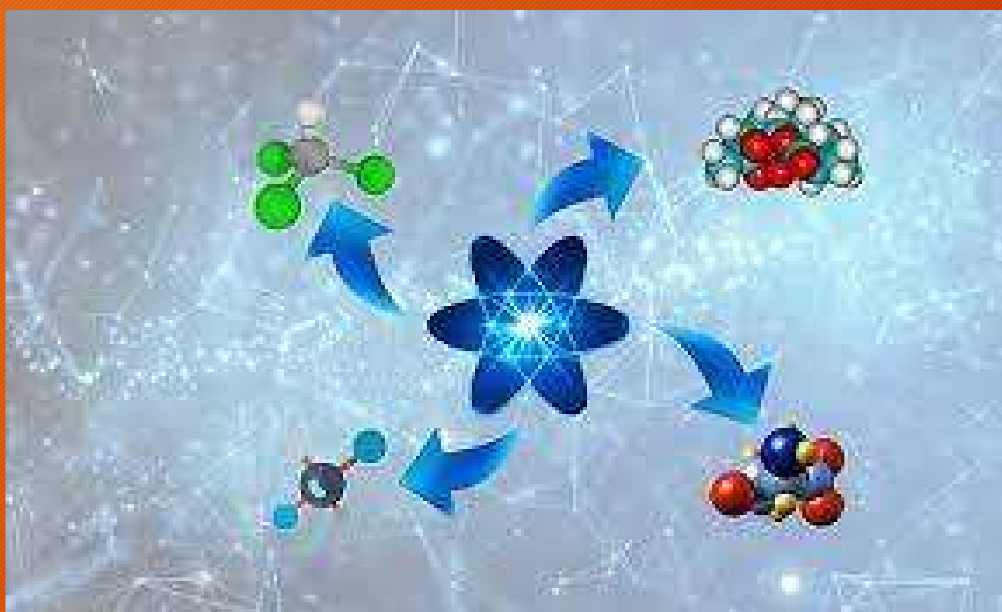
The stability of the hydrogen atom, is possible thanks to the Uncertainty Principle.



Bohr model is wrong

QUANTUM MECHANICS IN BIOLOGY

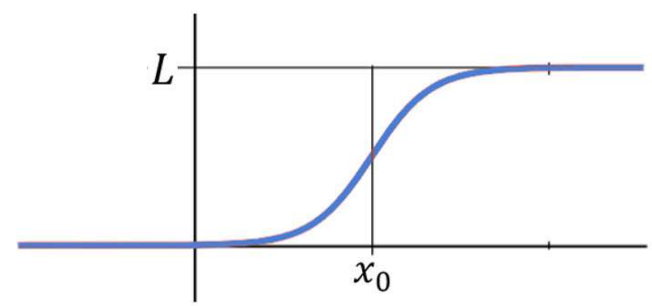
- Propagation of virus. Pandemic and others. We can construct field theories around this.



Logistic Function

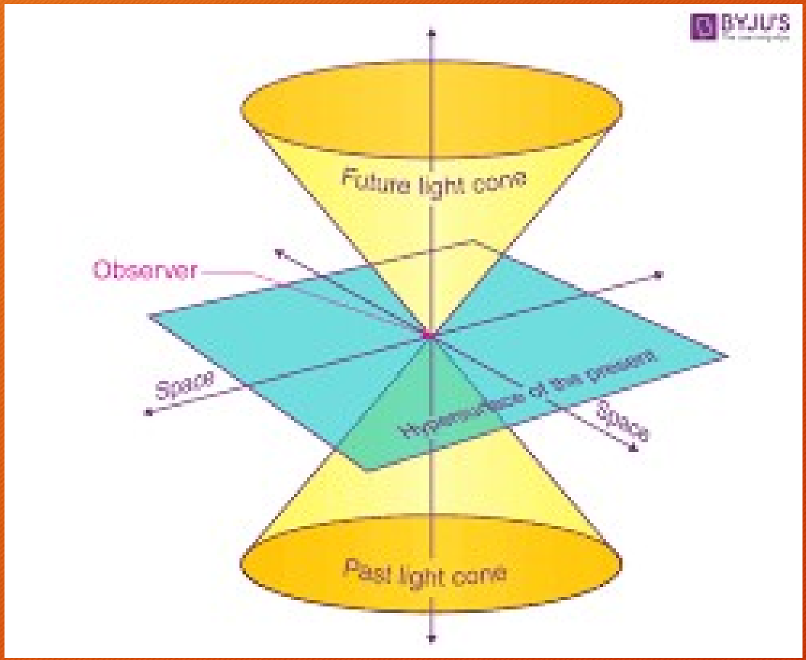
$$f(x) = \frac{L}{1 + e^{-k(x-x_0)}}$$

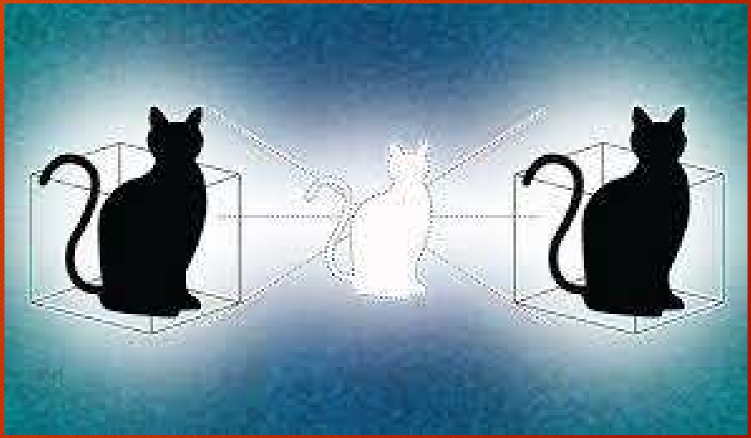
x_0 = x value of midpoint
L = maximum value
k = growth rate



I. Arraut, et. al, Epidemic Analytics for Decision Supports in COVID19 Crisis

QUANTUM MECHANICS+SPECIAL RELATIVITY





It was a complicated problem by the time.

WHY THE SINGLE PARTICLE APPROACH FAILS IN RELATIVITIC QUANTUM MECHANICS?

$$0 = (i\partial\!\!\!/ - m)\psi = (\partial^2 + m^2)\psi$$

$$\text{Dirac} \Rightarrow \text{KG}$$

Klein-Gordon: $$\eta^{ab}\partial_a\partial_b\psi - \frac{m^2c^2}{\hbar^2}\psi = 0$$

Minkowski metric; Second-order space and time

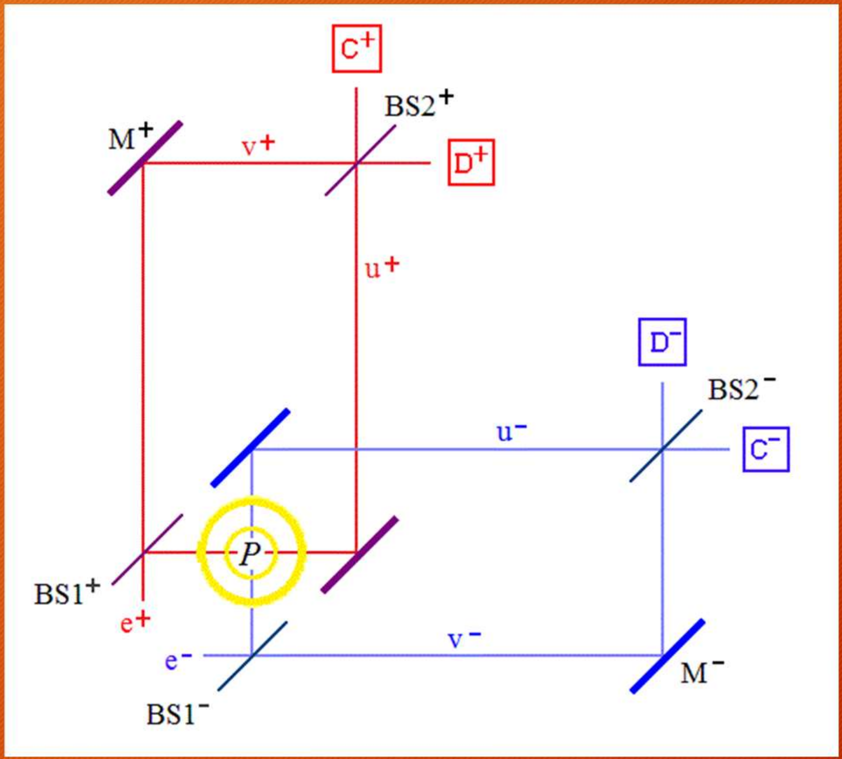
Schrödinger: $$\frac{\hbar^2}{2m}\nabla^2\psi + i\hbar\partial_t\psi = 0$$

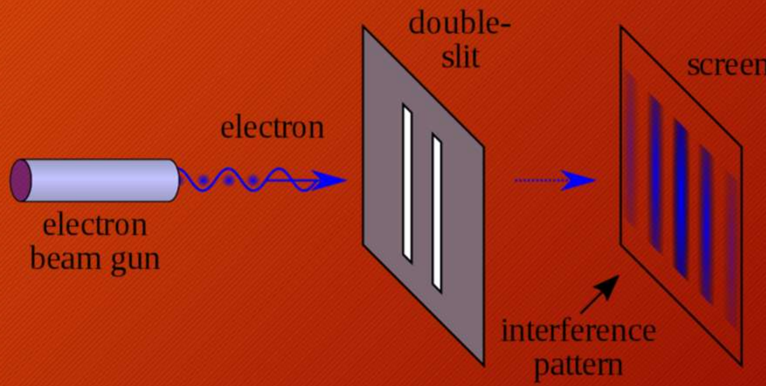
Second-order space; First-order time

Dirac: $$i\hbar\gamma^a\partial_a\psi - mc\psi = 0$$

Dirac spin matrix; First-order space and time

PARADOXES EMERGING FROM THE SINGLE PARTICLE APPROACH

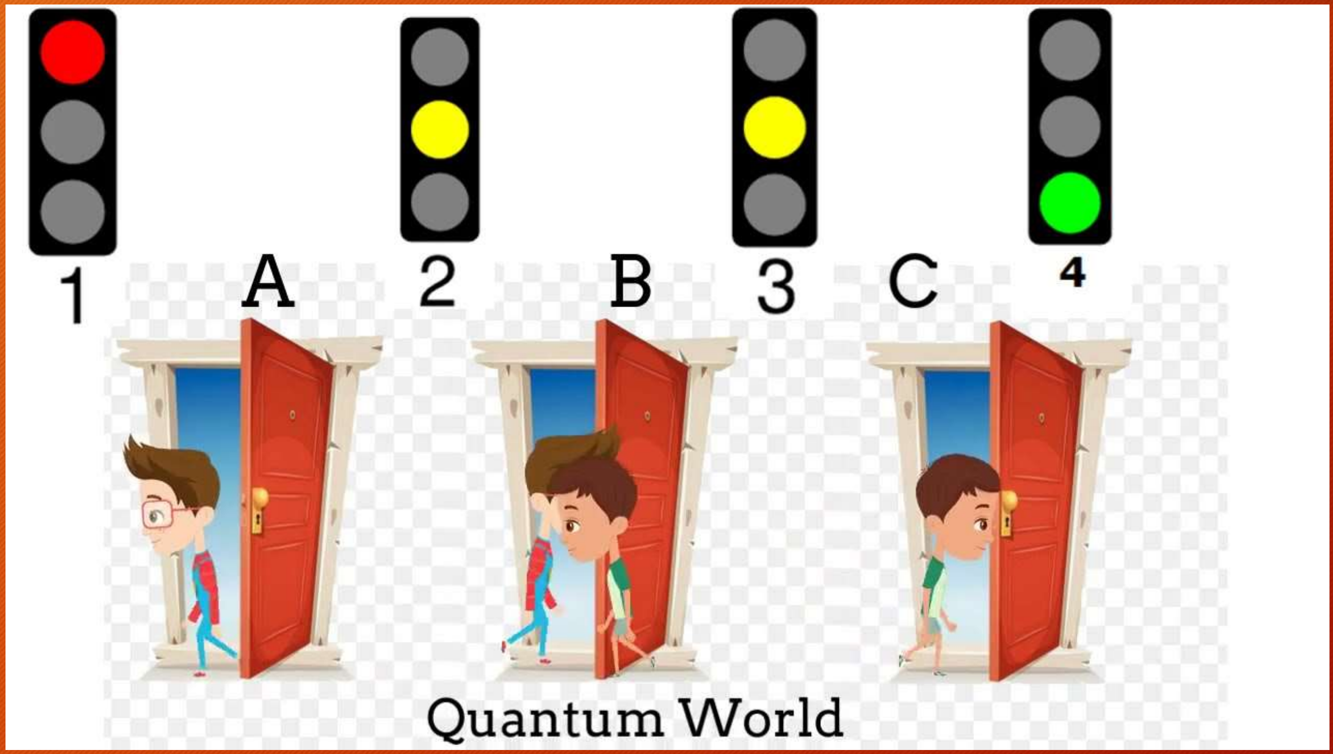




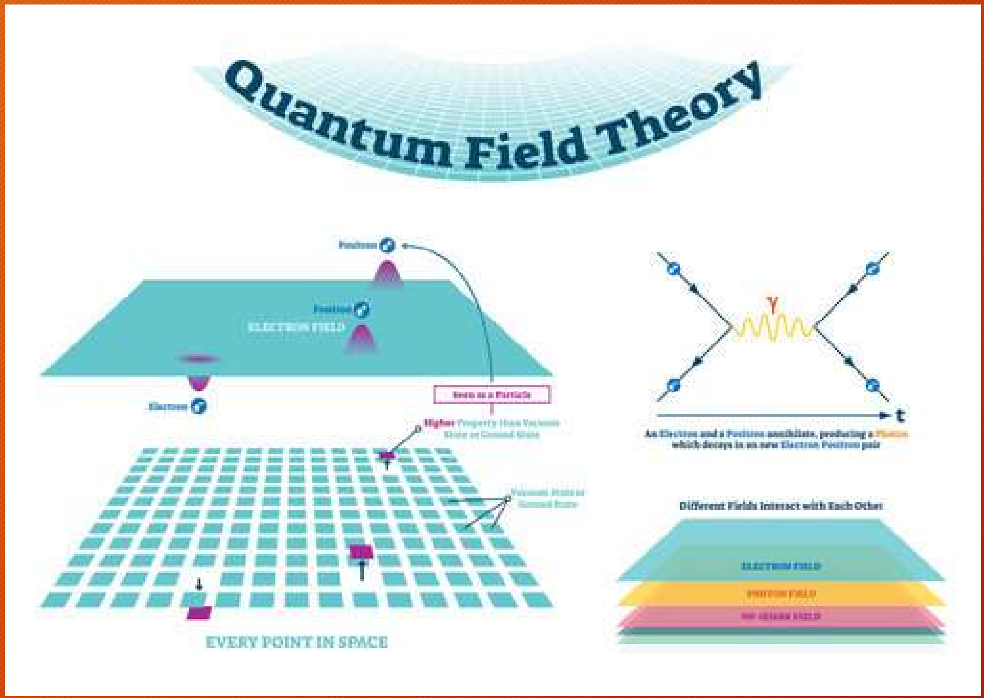
Wave-particle duality

I. Arraut, arXiv:2106.06397 **[physics.gen-ph]**

HARDY'S EXPERIMENT

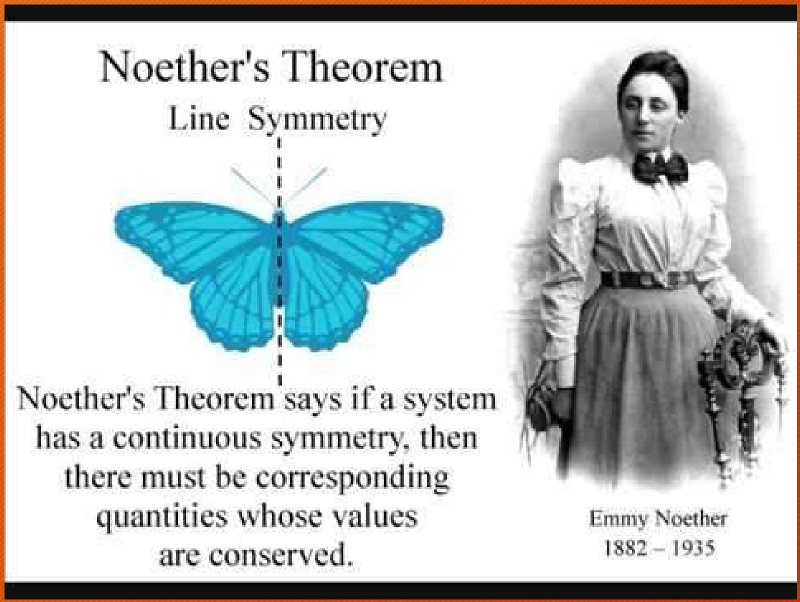


QUANTUM FIELD THEORY (MULTIPARTICLE APPROACH)



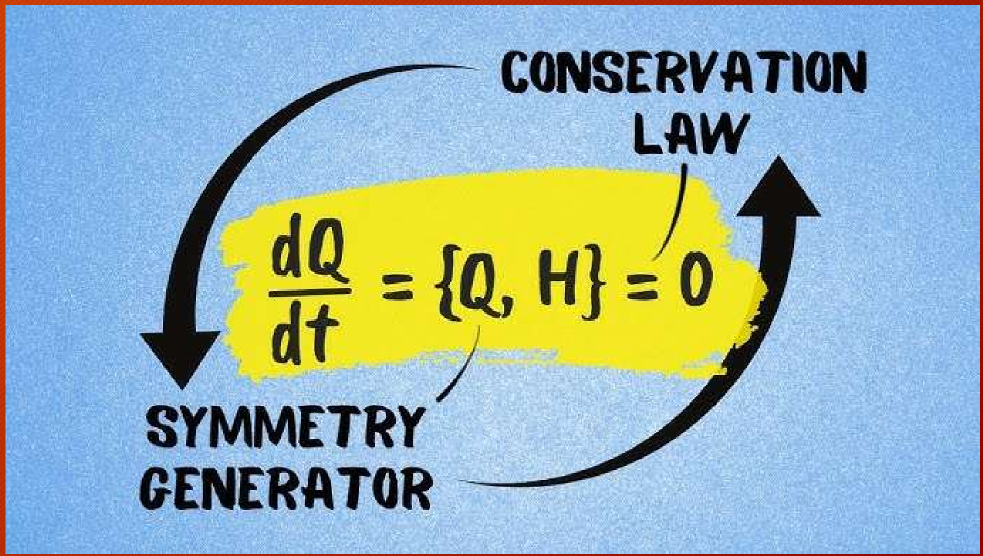
- Consistent formulation

THE ROLE OF SYMMETRY IN THE INTERACTIONS

- 

Important classically. At the Quantum level, interesting

- issues occur.



LAGRANGIAN VS HAMILTONIAN FORMULATION

Lagrangian

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = \frac{\partial L}{\partial \theta}$$

$$L = T - U$$

$$\frac{\partial L}{\partial q_i} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} = 0$$

Hamiltonian

$$H_{operator} = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)$$

Operator associated with kinetic energy

Potential energy

SOME BASIC OPERATORS

Position and momentum operators

Position operator $X = \int dx\, |x\rangle x \langle x|$

Momentum operator $P = \int dp\, |p\rangle p \langle p|$

Translation operator $T(\alpha)|x\rangle = |x + \alpha\rangle$

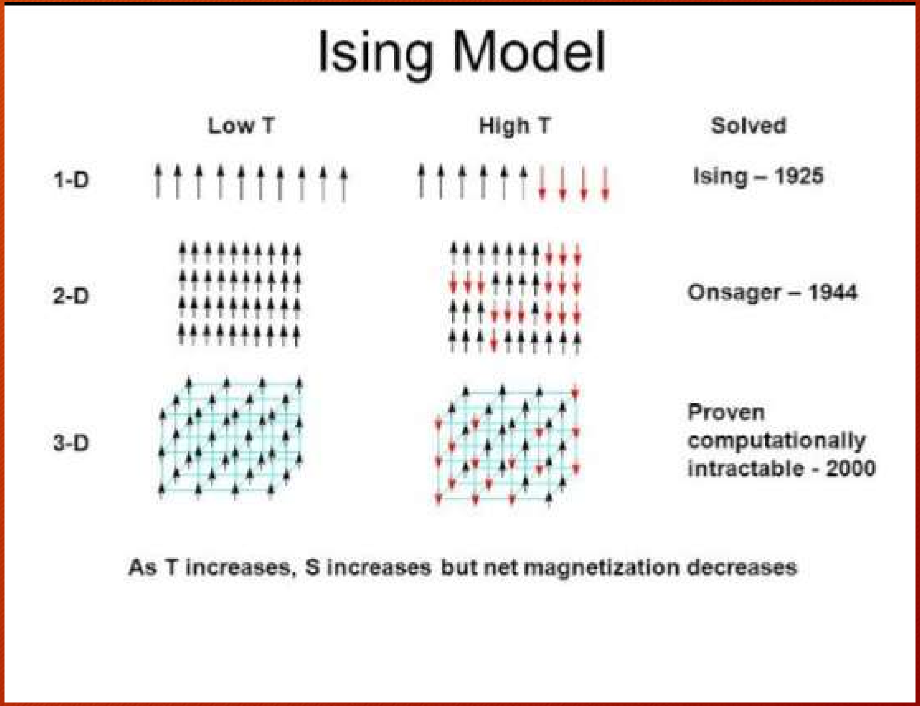
Representations

$$\langle x|P = -i\hbar \frac{\partial}{\partial x}\langle x| \qquad \langle p|X = i\hbar \frac{\partial}{\partial p}\langle p|$$

WHEN TO USE LAGRANGIAN OR HAMILTONIAN

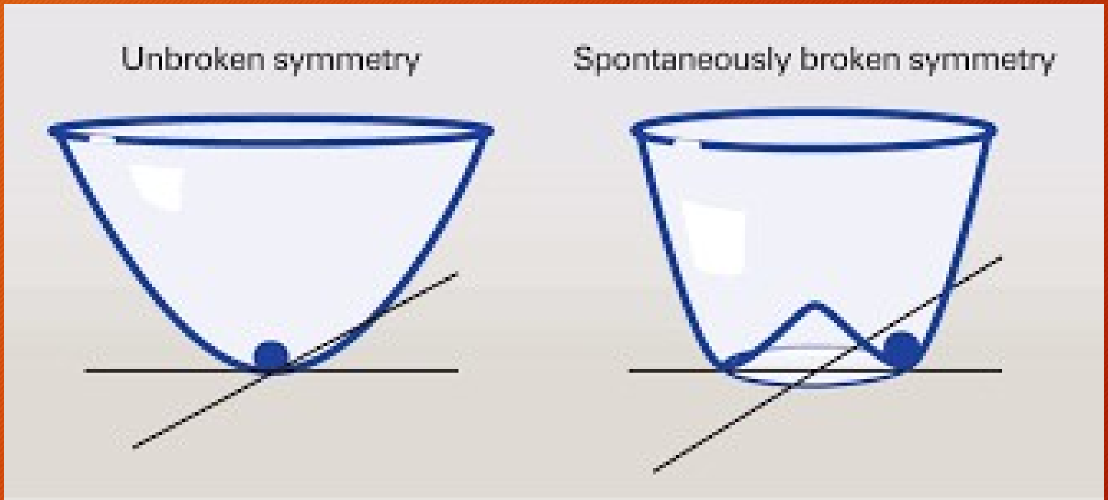
Use Lagrangian here

Use Hamiltonian here



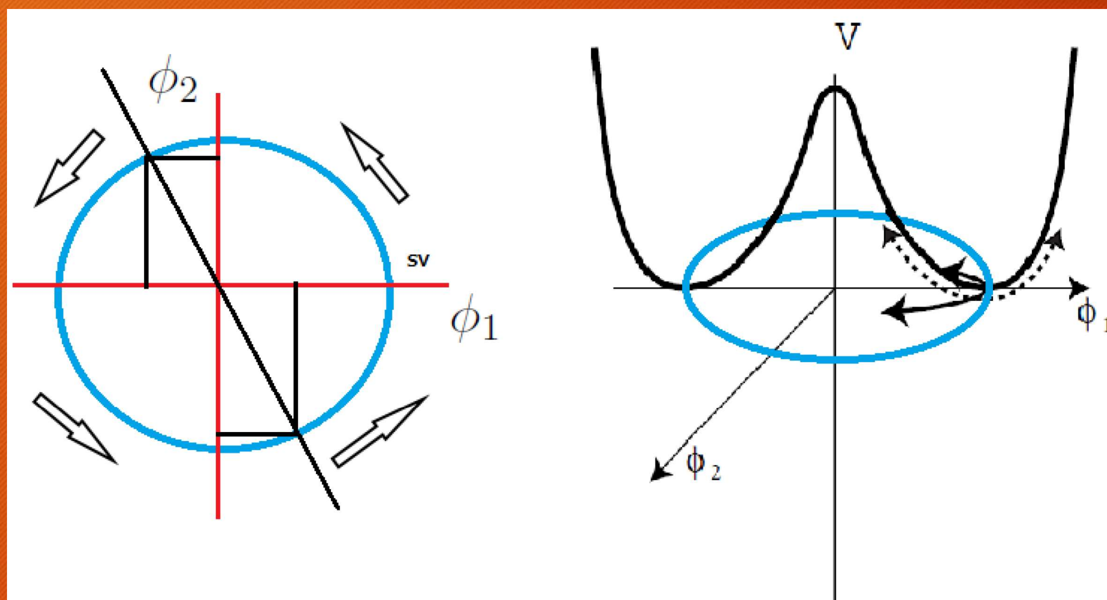
SPONTENAEOUS SYMMETRY BREAKING

- Symmetries of the Lagrangian satisfied. Symmetries of the ground state violated.

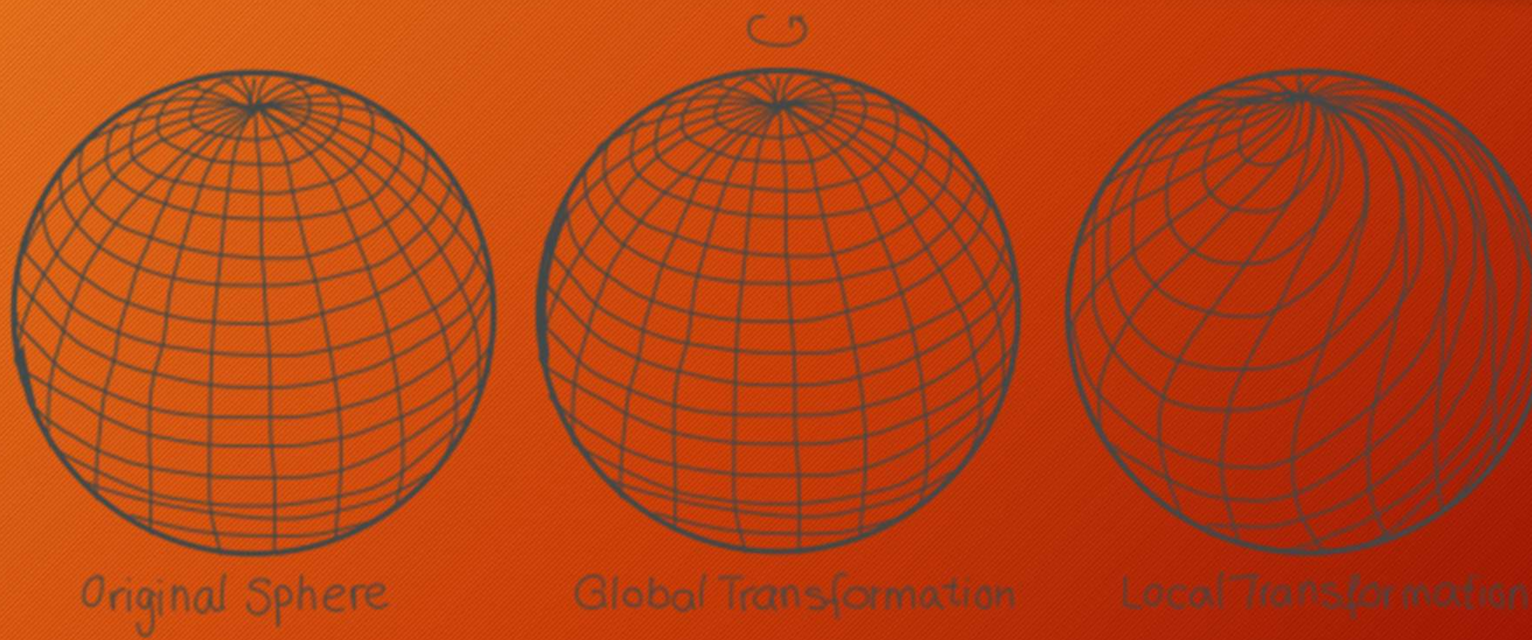


Broken symmetry operators. They map one vacuum state toward another one.

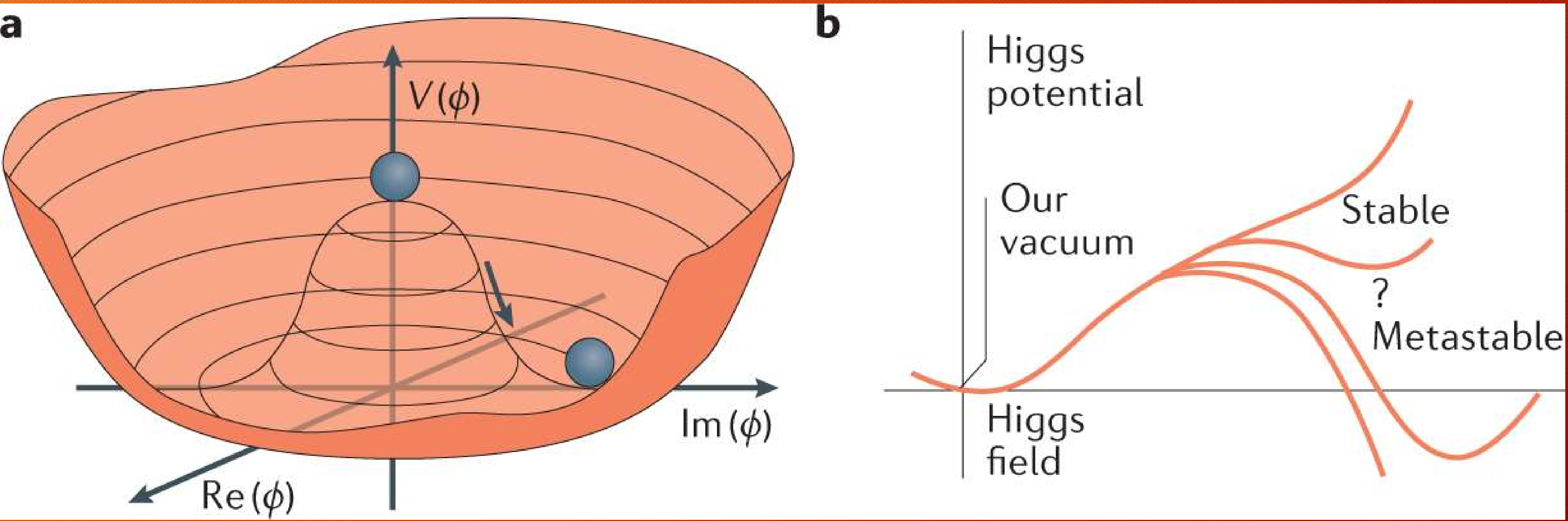
Nambu-Goldstone bosons



GLOBAL VS LOCAL SYMMETRIES



HIGGS MECHANISM IN QFT



GLOBAL VS LOCAL SYMMETRIES

- Global symmetries: Their transformation parameter does not depend on a local variable (like spacetime).

$$\mathcal{L} = (\partial_\mu \phi)(\partial^\mu \phi^*) - m^2 \phi^* \phi - \lambda(\phi^* \phi)^2$$
$$= (\partial_\mu \phi)(\partial^\mu \phi^*) - V(\phi, \phi^*).$$

Is invariant under

$$\phi \rightarrow e^{i\Lambda} \phi$$

$$\frac{\partial V}{\partial \phi} = m^2 \phi^* + 2\lambda \phi^* (\phi^* \phi)$$

Global symmetry

The vacuum is

$$|\phi|^2 = -\frac{m^2}{2\lambda} = a^2$$

The vacuum is single or degenerate depending on the values taken by the free-parameters

GROUP THEORY AND SSB

- We have a group of symmetries, which do not leave the vacuum invariant

$$G: \quad \phi_0' = U(g)\phi_0 \neq \phi_0;$$

$$H: \quad \phi_0' = U(h)\phi_0 = \phi_0$$

$$U(h) = e^{iT_3\alpha_3}.$$

→ Subgroup of symmetries leaving the ground state invariant

$$G/H$$

↓

The dimension of the coset tells us about
The number of broken generators
(number) of Nambu-Goldstone bosons

HOW TO VISUALIZE THE NAMBU-GOLDSTONE FIELD IN THE LAGRANGIAN

- On the previous Lagrangian, make the replacement

$$\phi(x) = a + \frac{\phi_1(x) + i\phi_2(x)}{\sqrt{2}}$$

$$\langle \phi_1 \rangle_0 = \langle \phi_2 \rangle_0 = 0$$

Vacuum conditions

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu \phi_1)^2 + \tfrac{1}{2}(\partial_\mu \phi_2)^2 - 2\lambda a^2 \phi_1^2 - \sqrt{2}\lambda \phi_1(\phi_1^2 + \phi_2^2) - \frac{\lambda}{4}(\phi_1^2 + \phi_2^2)^2.$$

Can you identify which the massive and the massless terms?

LOCAL SYMMETRIES

- Consider now a local symmetry transformation as follows:

$$\phi \rightarrow e^{i\Lambda(x)}\phi.$$

- The only way to satisfy this symmetry is by adding a new (gauge) field which restore the symmetry locally

$$\mathcal{L} = (\partial_\mu + ieA_\mu)\phi(\partial^\mu - ieA^\mu)\phi^* - m^2\phi^*\phi - \lambda(\phi^*\phi)^2 - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}.$$

- Repeating the same processes as before:

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + e^2a^2A_\mu A^\mu + \tfrac{1}{2}(\partial_\mu\phi_1)^2 + \tfrac{1}{2}(\partial_\mu\phi_2)^2 - 2\lambda a^2\phi_1^2 + \sqrt{2}eaA^\mu\partial_\mu\phi_2 + \text{cubic} + \text{quartic terms}$$

LOCAL SYMMETRIES

- After the following transformation:

$$\phi_1' = \phi_1 - \Lambda\phi_2,$$

$$\phi_2' = \phi_2 + \Lambda\phi_1 + \sqrt{2}\Lambda a.$$

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + e^2a^2A_\mu A^\mu + \tfrac{1}{2}(\partial_\mu\phi_1)^2 - 2\lambda a^2\phi_1^2 + \text{coupling terms.}$$

Kinetic terms of the gauge field

Massive term of the gauge field

APPLICATIONS

- 1). SUPERCONDUCTIVITY: Consider the previous Lagrangian, but static with $\partial_0\phi = 0$

$$-\mathcal{L} = \tfrac{1}{2}(\nabla \times \mathbf{A})^2 + |(\nabla - ie\mathbf{A})\phi|^2 + m^2|\phi|^2 + \lambda|\phi|^4.$$

$$m^2 = a(T - T_c)$$

ϕ → Macroscopic many-particle wavefunction

$T > T_c$ → $m^2 > 0$ → $|\phi| = 0$

$T < T_c$ → $m^2 < 0$ → $|\phi|^2 = -\dfrac{m^2}{2\lambda} > 0;$

APPLICATIONS

We then have an invariance with respect to the following phase transformation: $$\phi \to e^{i\Lambda(x)}\phi, \quad \mathbf{A} \to \mathbf{A} + \frac{1}{e}\nabla\Lambda(x)$$

With the conserved current

$$\mathbf{j} = -i(\phi^*\nabla\phi - \phi\nabla\phi^*) - 2e|\phi|^2\mathbf{A}.$$

When $T < T_c$: → The second term dominates → $\mathbf{j} = \frac{em^2}{\lambda}\mathbf{A} = -k^2\mathbf{A}$

LONDON EQUATION FOR SUPERCONDUCTIVITY

HIGGS MECHANISM IN SUPERCONDUCTIVITY

$$\nabla \times \mathbf{B} = \mathbf{j}.$$

$$\nabla^2 \mathbf{B} = k^2 \mathbf{B}.$$

$$B_x = B_0 \, e^{-kx},$$

$$\Box A_\mu = -k^2 A_\mu$$

The photon became massive!!! Meissner effect!!!

ELECTROWEAK THEORY: UNIFICATION OF INTERACTIONS

$$\mathcal{L} = \mathrm{i}\bar{R}\gamma\cdot\partial R + \mathrm{i}\bar{L}\gamma\cdot\partial L$$

$$L \rightarrow \mathrm{e}^{-(\mathrm{i}/2)\tau\cdot\alpha} L,$$

$$R \rightarrow R,$$

$$SU(2): \begin{pmatrix} \nu_e \\ e_\mathrm{L} \\ e_\mathrm{R} \end{pmatrix} \rightarrow \left(\begin{array}{cc|c} \mathrm{e}^{-(\mathrm{i}/2)\tau\cdot\alpha} & & 0 \\ & & 0 \\ \hline 0 & 0 & 1 \end{array}\right) \begin{pmatrix} \nu_e \\ e_\mathrm{L} \\ e_\mathrm{R} \end{pmatrix}.$$

$$U(1): \begin{pmatrix} \nu_e \\ e_\mathrm{L} \\ e_\mathrm{R} \end{pmatrix} \rightarrow \begin{pmatrix} \mathrm{e}^{in\beta} & 0 & 0 \\ 0 & \mathrm{e}^{in\beta} & 0 \\ 0 & 0 & \mathrm{e}^{\mathrm{i}\beta} \end{pmatrix} \begin{pmatrix} \nu_e \\ e_\mathrm{L} \\ e_\mathrm{R} \end{pmatrix}$$

Transformation over the singlet

ELECTROWEAK UNIFICATION

Gauging $SU(2)$

$$D_\mu L = \partial_\mu L - \frac{i}{2} g \boldsymbol{\tau} \cdot \mathbf{W}_\mu L.$$

Covariant derivative

Gauging $U(1)$

$$D_\mu L = \partial_\mu L + \frac{i}{2} g' X_\mu L,$$

$$D_\mu R = \partial_\mu R + i g' X_\mu R.$$

The Lagrangian then becomes:

$$\mathcal{L}_1 = i\bar{R}\gamma^\mu(\partial_\mu + ig' X_\mu)R + i\bar{L}\gamma^\mu\left(\partial_\mu + \frac{i}{2}g' X_\mu - \frac{i}{2} g \boldsymbol{\tau} \cdot \mathbf{W}_\mu\right)L$$
$$- \tfrac{1}{4}(\partial_\mu \mathbf{W}_\nu - \partial_\nu \mathbf{W}_\mu + g\mathbf{W}_\mu \times \mathbf{W}_\nu)^2 - \tfrac{1}{4}(\partial_\mu X_\nu - \partial_\nu X_\mu)^2.$$

Now we introduce the Higgs field

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}.$$

→

$$D_\mu \phi = \left(\partial_\mu - \frac{i}{2} g \boldsymbol{\tau} \cdot \mathbf{W}_\mu - \frac{i}{2} g' X_\mu\right)\phi.$$

→

$$\mathcal{L}_2 = (D_\mu \phi)^\dagger (D_\mu \phi) - \frac{m^2}{2}\phi^\dagger \phi - \frac{\lambda}{4}(\phi^\dagger \phi)^2 - G_e(\bar{L}\phi R + \bar{R}\phi^\dagger L).$$

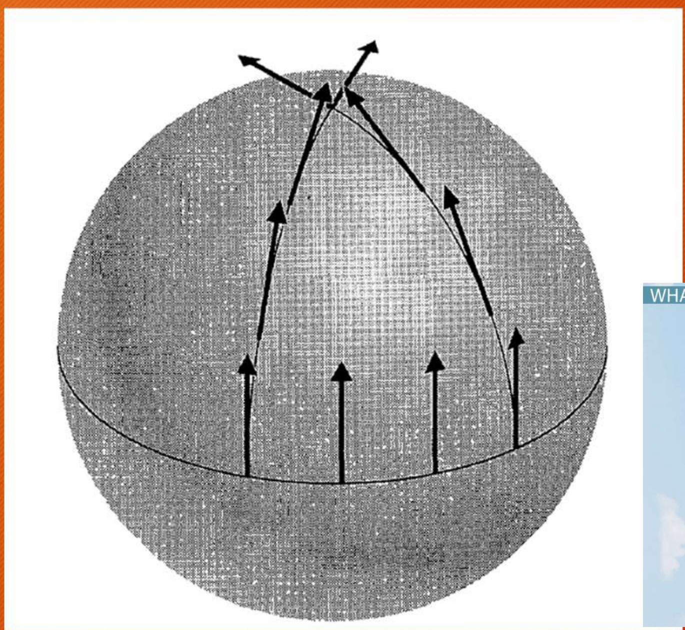
GAUGE THEORY IN GRAVITY

- General Relativity is a gauge theory

$$G_{\mu\nu} = \kappa T_{\mu\nu}$$

$$\nabla_\mu V^\nu = \partial_\mu V^\nu + \Gamma^\nu_{\mu\sigma} V^\sigma$$ → Covariant derivative

$$\Gamma^\sigma_{\mu\nu} = \frac{1}{2} g^{\sigma\rho}(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu}).$$ → Christoffel connections



Gauge theory showing the local equivalence of two theories

Hamiltonians in the stock market: The Black Scholes Hamiltonian (B. E. Baaquie, Cambridge University Press (2004), pp 52-75.)

- Free parameters: Volatility and interest rate

$$\frac{\partial \psi}{\partial t} = \hat{H}_{BS}\psi,$$

↓

$$\hat{H}_{BS} = -\frac{\sigma^2}{2}\frac{\partial^2}{\partial x^2} + \left(\frac{1}{2}\sigma^2 - r\right)\frac{\partial}{\partial x} + r.$$

The Black Scholes Hamiltonian

$$\Pi = \psi - \frac{\partial \psi}{\partial S} S.$$

$$\frac{d\Pi}{dt} = \frac{\partial \psi}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 \psi}{\partial S^2}.$$

$$\frac{d\Pi}{dt} = r\Pi.$$

Eliminating randomness

$$\frac{\partial \psi}{\partial t} + rS\frac{\partial \psi}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 \psi}{\partial S^2} = r\psi.$$

$S = e^x$, where $-\infty < x < \infty$.

F. Black and M. Scholes, J. Pol. Ec., 81 (1973), p. 637; Journ. Pol. Ec. 81 (1973):637

The Black Scholes solutions

$$c_t = S_t N(d_1) - K\, e^{-r(T-t)} N(d_2),$$

$$p_t = K\, e^{-r(T-t)} N(-d_2) - S_t N(-d_1)$$

c_t = Price of a European call option at time t;
p_t = Price of a European put option at time t;
S_t = Price of the underlying stock at time t;
$T - t$ = Time to maturity;
K = Strike price of the option;
σ = Volatility of the underlying stock;
r = Interest rate.

Call and Put Options prices.

$$d_1 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

Within the Hamiltonian formulation, we do not need these details for understanding the dynamic of the system.

The Merton-Garman Hamiltonian

$$S = e^x, \qquad -\infty < x < \infty,$$
$$\sigma^2 = V = e^y, \qquad -\infty < y < \infty,$$

$$\frac{\partial C}{\partial t} = \hat{H}_{MG} C,$$

$$\hat{H}_{MG} = -\frac{e^y}{2}\frac{\partial^2}{\partial x^2} - \left(r - \frac{e^y}{2}\right)\frac{\partial}{\partial x} - \left(\lambda e^{-y} + \mu - \frac{\zeta^2}{2} e^{2y(\alpha-1)}\right)\frac{\partial}{\partial y} - \rho\zeta e^{y(\alpha-1/2)}\frac{\partial^2}{\partial x \partial y} - \zeta^2 e^{2y(\alpha-1)}\frac{\partial^2}{\partial y^2} + r.$$

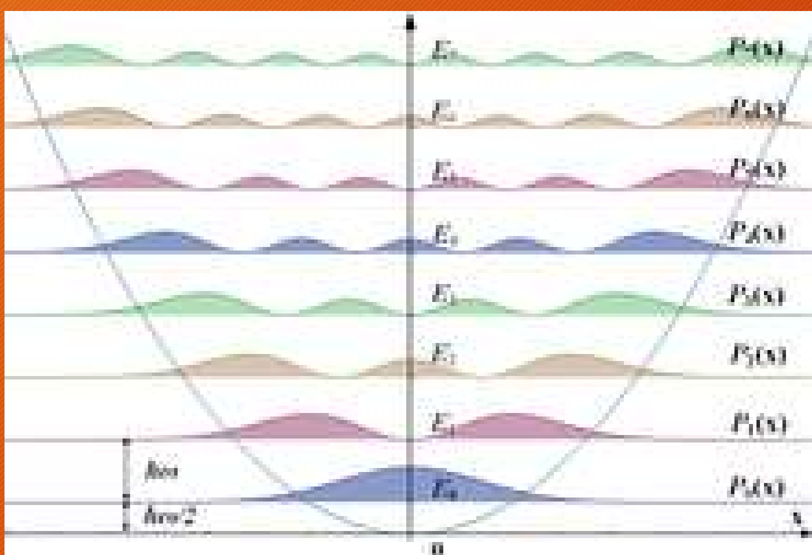
Here the price of the Option as well as the price of the volatility are stochastic variables

R. C. Merton, Bell Journ. Econ. Manag. Sc. 4 (1973):141-183.

Equilibrium (Vacuum) condition for the stock market: The martingale state

$$\hat{H}|S>=0.$$

The vacuum state is annihilated by the Hamiltonian



The vacuum of a Hamiltonian is normally the state at the Bottom. It is equivalent to the martingale condition in the market.

(Peskin and Schroeder, An Introduction to Quantum Field Theory, Taylor and Francis; Arraut et.al, *EPL* 131 (2020) 6, 68003).

The influence of the free-parameters on the dynamic of the system (Black Scholes and Merton-Garman)

Sometimes changing the free-parameters of a system, changes the vacuum structure. Symmetries which were perfect, become spontaneously broken. This occurs in physics as well as in Finance and other areas of fundamental science.

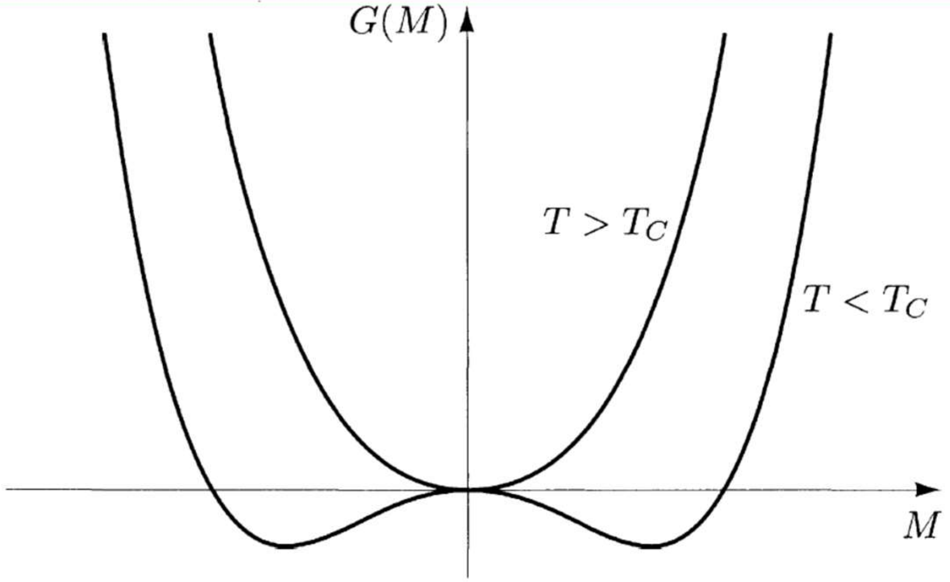


Figure 8.3. Behavior of the Gibbs free energy $G(M)$ in Landau theory, at temperatures above and below the critical temperature.

Figure taken from Peskin and Schroeder

Example of Spontaneous symmetry breaking

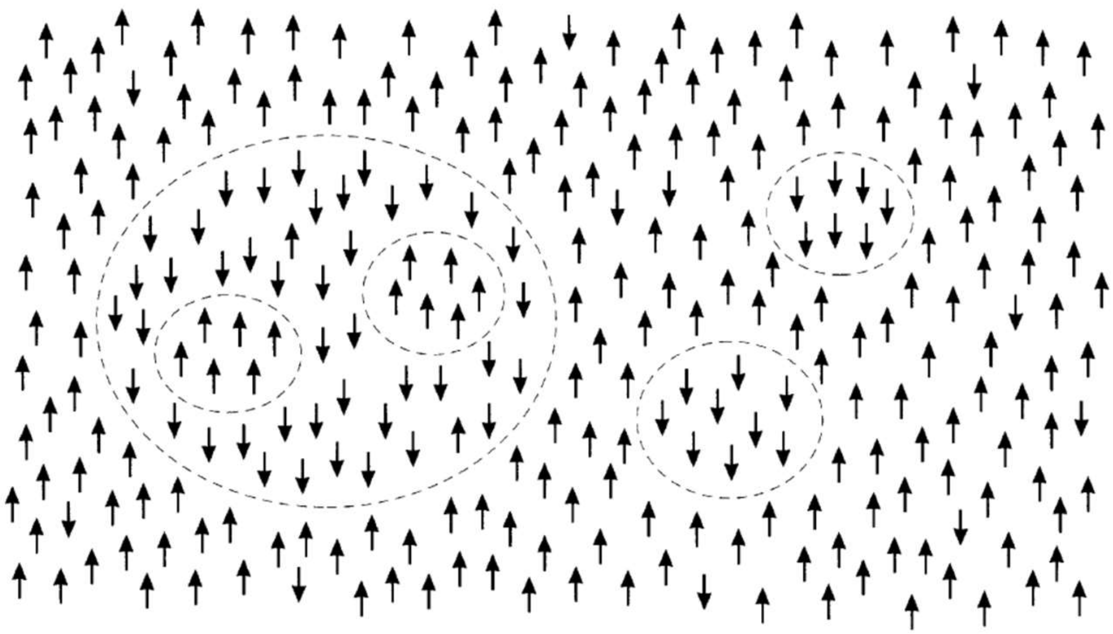


Figure 8.1. Clusters of oriented spins near the critical point of a ferromagnet.

Figure taken from Peskin and Schroeder

$$\hat{p}|S> \neq 0.$$

When a symmetry is spontaneously broken, its operator does not annihilate the vacuum State.

CONSEQUENCES OF SSB IN THE STOCK MARKET

$$\hat{p}_x|S> \neq 0, \quad \hat{p}_y|S> \neq 0.$$

In the MG equation, symmetries under
Changes of price and volatility are SSB.

$$\phi_{yvac} = \left(\frac{\lambda e^{-y} + \mu - \frac{\zeta^2}{2} e^{2y(\alpha-1)}}{r - \frac{e^y}{2}} \right) \phi_{xvac},$$

Compromise between prices of the Option and volatility

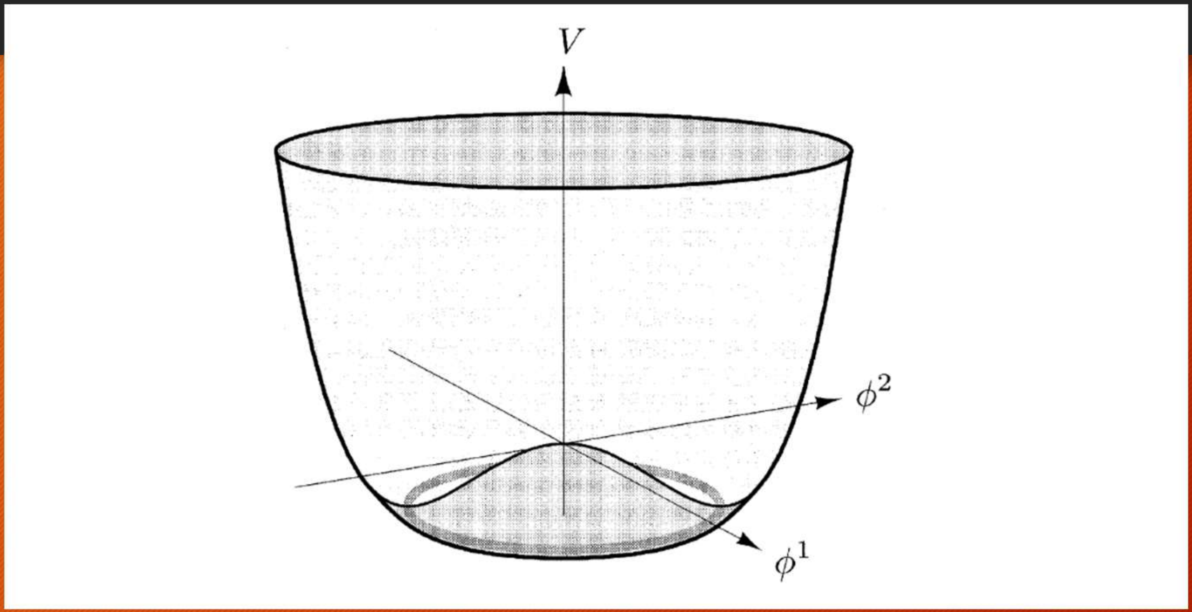
$$\hat{p}|S> \neq 0.$$

In the BS equation, since the volatility
is a constant, then only the symmetry under
Changes of prices is SSB.

$$< x, y|S >= S(x, y, t) = e^{x+y} = \sum_{n=0}^{\infty} \phi_x^n \phi_y^n$$

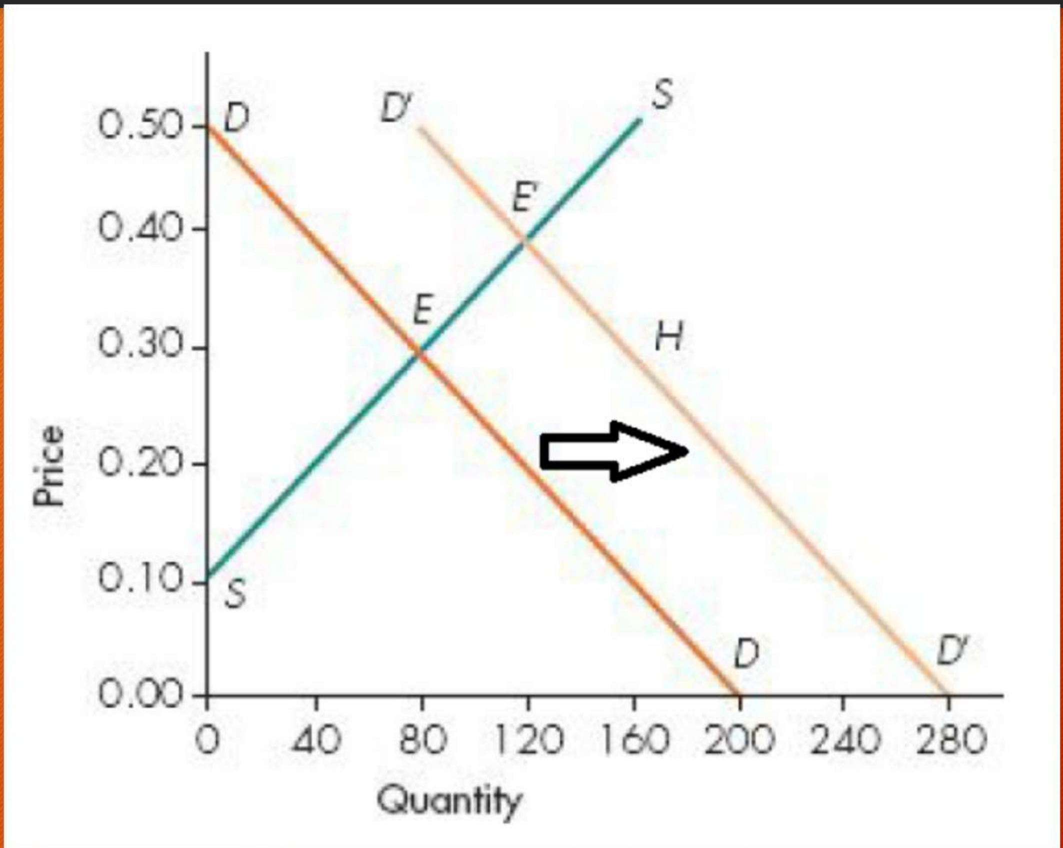
I. Arraut, et.al, EPL, 131 (2020) 68003

SPONTANEOUS SYMMETRY BREAKING



The famous Mexican hat potential!!! Each point along the minimal is a different vacuum state living in a different Hilbert space. Each vacuum is orthogonal to each other!!!

Market equilibrium in the stock market: Hamiltonian formulation



Demand and supply curve.
Begg D., et. al, "Economics"
McGraw-Hill Education

$$Q^D = a - bP,$$

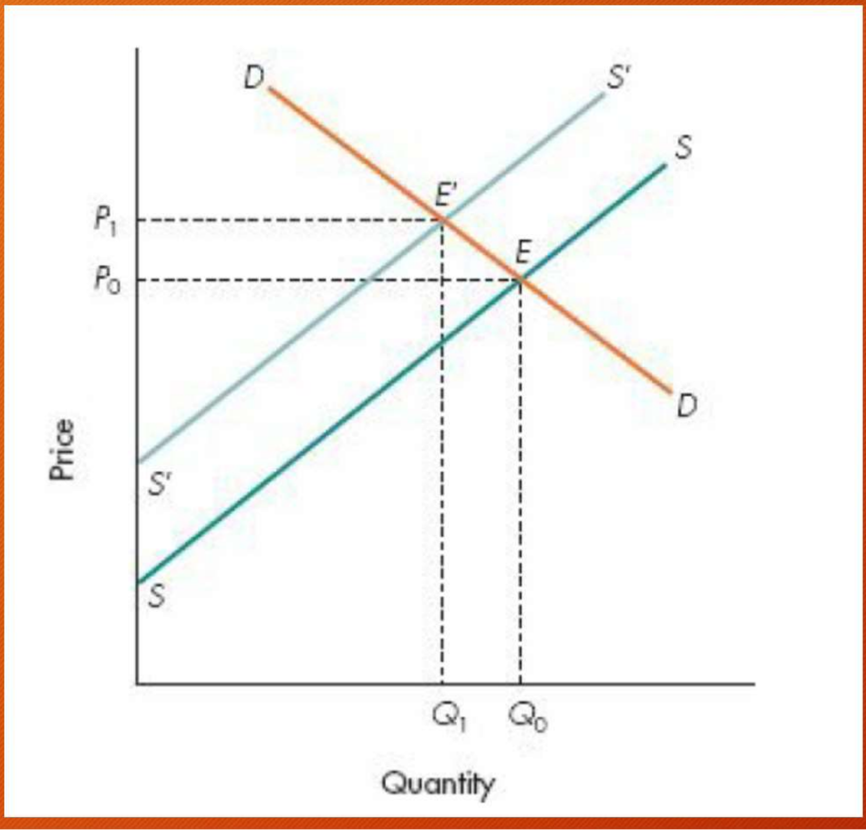
$$Q^S = c + dP,$$

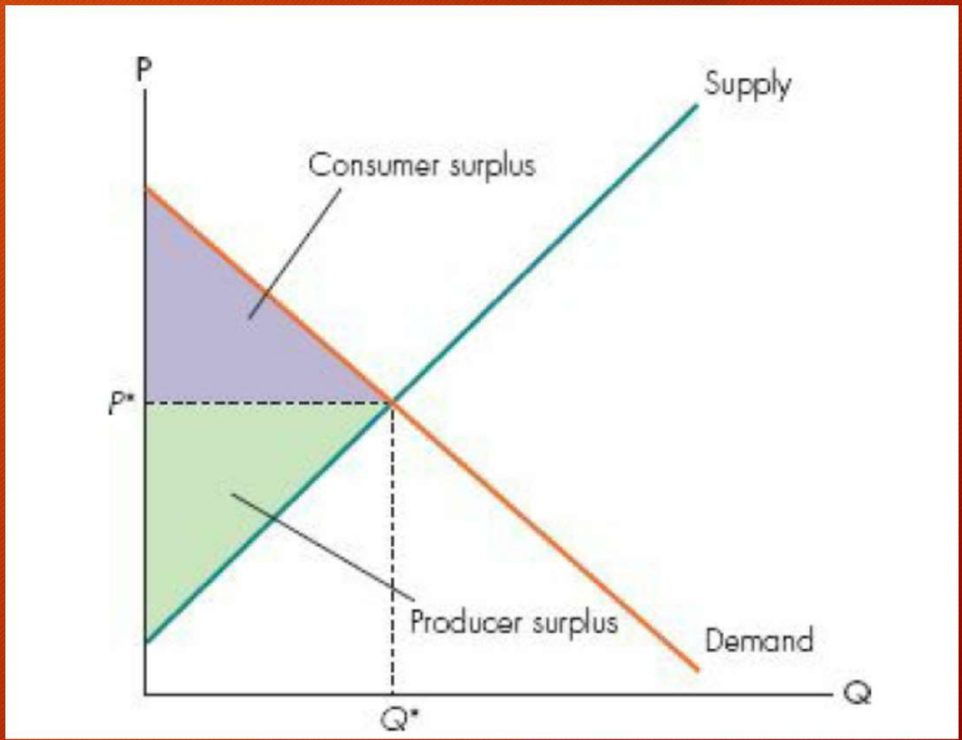
Market equilibrium occurs when:

$$Q^D = Q^S$$

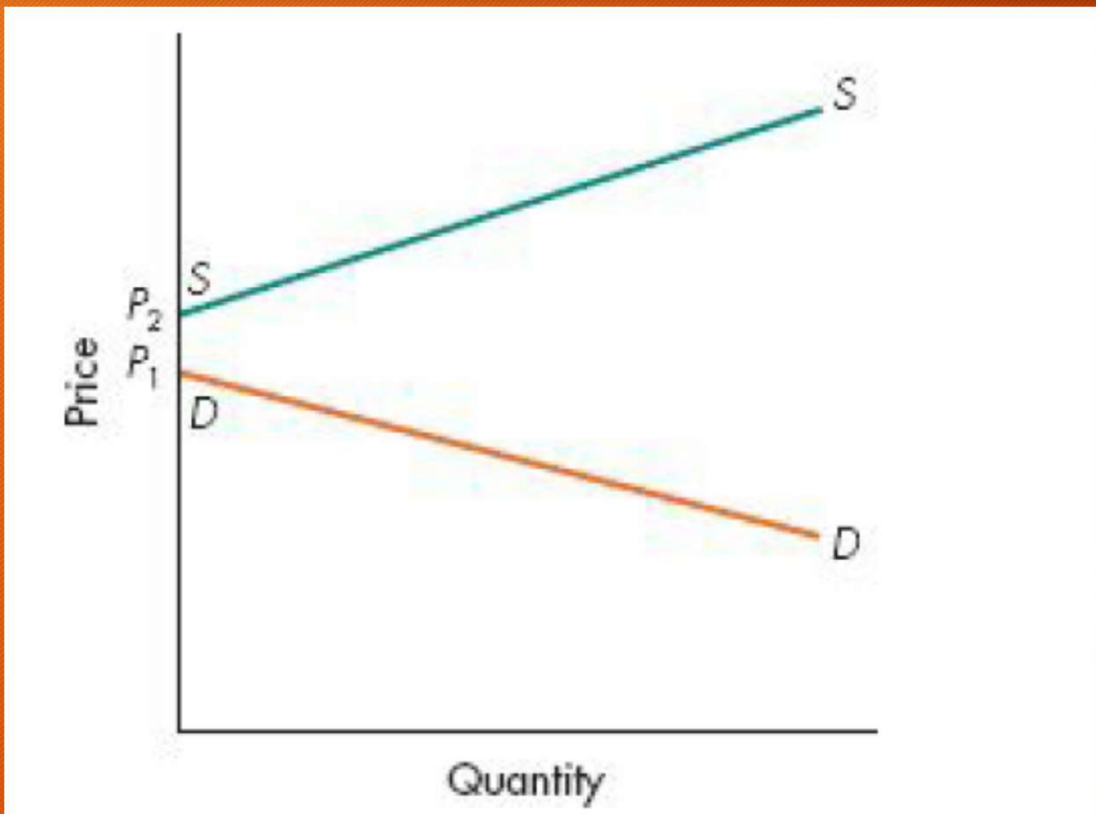
Supply and Demand

Demand and supply curve.
Begg D., et. al, “Economics”
McGraw-Hill Education





DISAGREEMENT IN PRICES BETWEEN SUPPLIERS AND CUSTOMERS



Demand and supply curve.
Begg D., et. al, “Economics”
McGraw-Hill Education

SUPPLY AND DEMAND: Accelerated particle and Harmonic behavior.

Hamiltonians describing the demand and supply behavior:

$$H_D = \frac{P_D^2}{2m_D} + kQ_D^2.$$

$$H_S = \frac{P_S^2}{2m_S} - \kappa Q_S^2.$$

Agreement in prices give:

$$H_S m_S + \kappa m_S Q_S^2 = H_D m_D - k m_D Q_D^2.$$

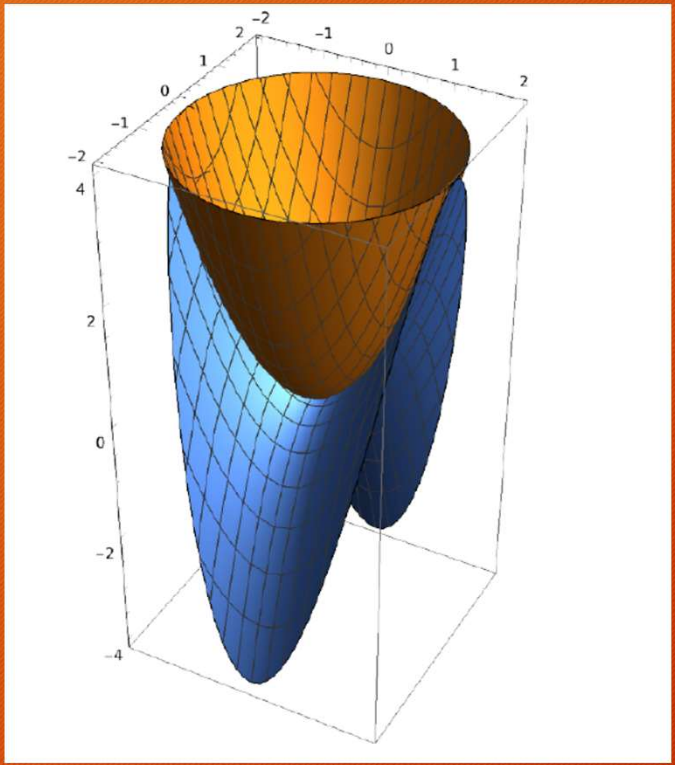
I. Arraut, W. Rosado and V. Leong, Mathematics 12 (6), 847.

Total equilibrium gives:

$$Q^* = \sqrt{\frac{H_D m_D - H_S m_S}{\kappa m_S + k m_D}}.$$

$$P^* = \sqrt{2\left(\frac{k\hat{H}_S m_S + \kappa \hat{H}_D m_D}{\kappa + k}\right)}.$$

SUPPLY AND DEMAND GAME: ZERO SUM GAME?



The general behavior of both Hamiltonians when plotted together.

The merge of two systems (Supply-demand): Market equilibrium

A SINGLE Hamiltonian can help to merge the supply and demand curve into a single system as:

$$H = \frac{P^2}{2m} + aQ^2 + bQ^4.$$

⬇

$$a = H_S - H_D, \qquad b = \frac{k + \kappa}{2}.$$

The equilibrium here is defined as the ground state (vacuum) of the system by considering the potential $V(x) = aQ^2 + bQ^4.$

MERGING SUPPLY AND DEMAND SYSTEMS

- The ground state is defined as

$$Q^{*2} = -\frac{a}{2b}.$$

$a > 0$ → Unbroken symmetry and trivial result

$a < 0$ → Broken symmetry and a non-trivial Equilibrium. SSB.

Paper in process of submission.

Game Theory: Decision trees

- Until now we have seen different tools of physics. Mainly those related to the variational principle (Lagrangian and Hamiltonian). As a final example, the final tool to consider occurs in Decision Trees when there are multiple decisions taken simultaneously.

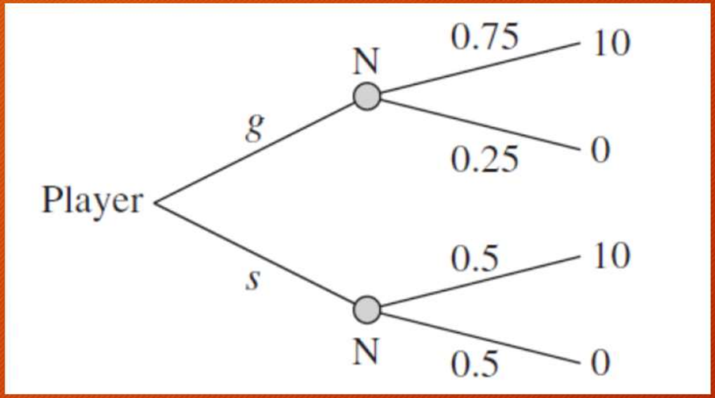
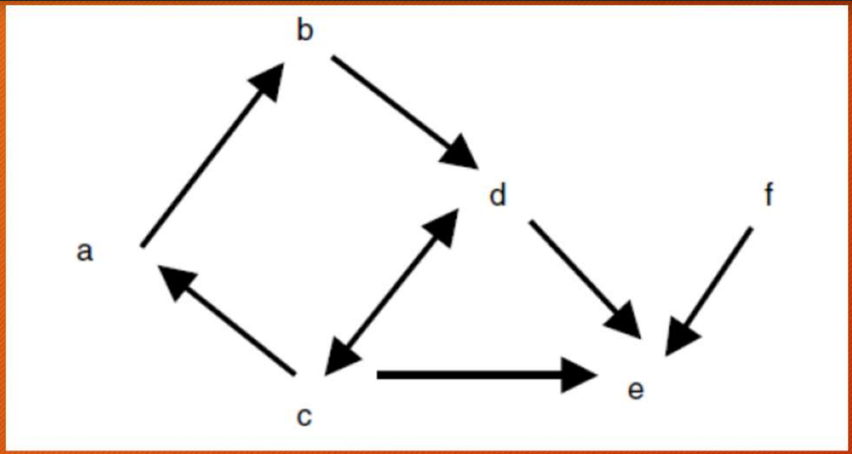
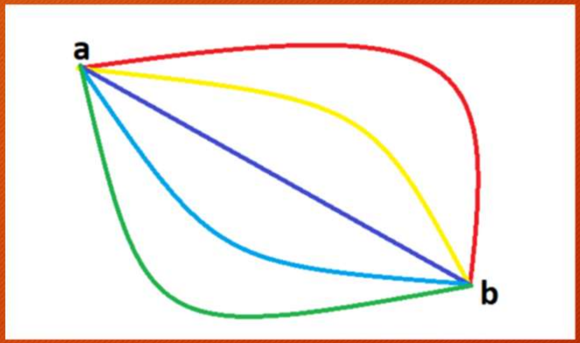


Figure taken from:
Tadelis, S., Game Theory: An Introduction, Princeton University Press, 2013

The Quantum Yang Baxter Equations

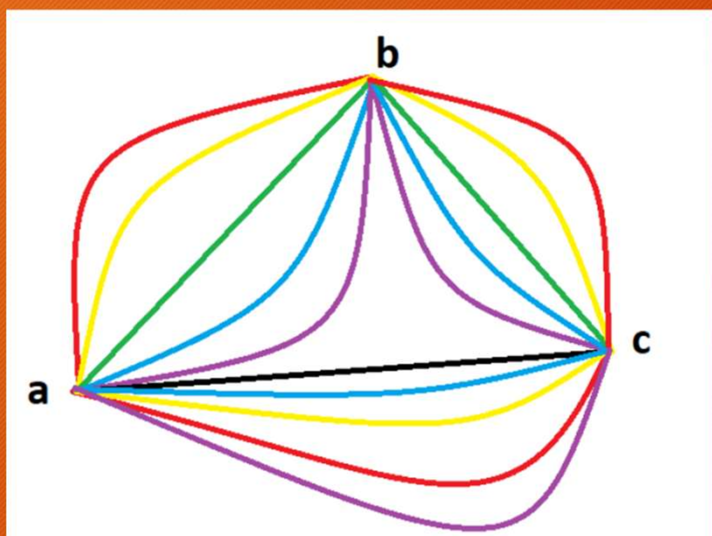


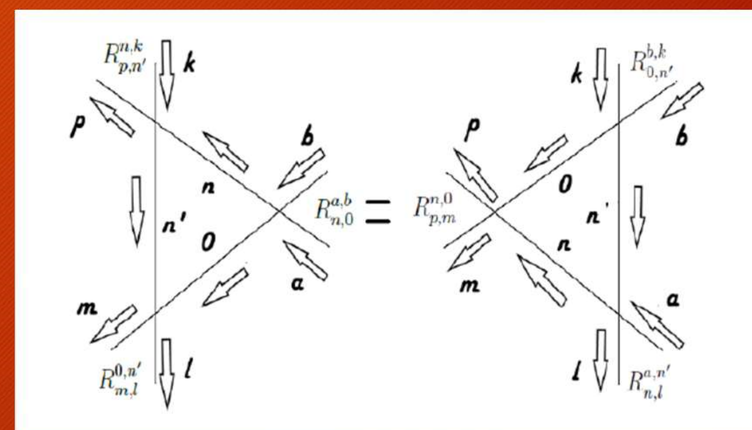
Kelly, A., Decision making using Game theory: An introduction for managers; Cambridge University Press 2003.



How to deal with multiple-decisions taken simultaneously and in a triangular form.

QYBE





$$R_{(1,2)}R_{(1,3)}R_{(2,3)} = R_{(2,3)}R_{(1,3)}R_{(1,2)}.$$

$$R^{0,n'}_{m,l}R^{n,k}_{p,n'}R^{a,b}_{n,0} = R^{n,0}_{p,m}R^{a,n'}_{n,l}R^{b,k}_{0,n'}.$$

This is just a minor example about how to deal with nodes connected as a triangle.

CONCLUSIONS

- 1). The tools of physics are very useful for analyzing practical problems.
- 2). The variational principle (Hamiltonian and Lagrangian formulation) is the most useful way to analyze the dynamic of a system without solving the involved equations.
- 3). Different values of the free parameters of the system involve different behaviors for the dynamic of the system. Particularly for its ground state.